Math 53 Homework 4

Due Wednesday 2/17/16 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 2/8 – Velocity, acceleration

- **Read:** sections 13.2; 13.3 to middle of p. 863; 13.4 to p. 873.[1]
- **Work:** 13.2: (41), (47), $\underline{53}$, $\underline{55}$, $\underline{56}$.[2]

  13.4: (3), $\underline{10}$, (15).
- **Bonus problem** (extra credit, hard): Problem 2 below. (next page)

Wednesday 2/10 – Functions of several variables

- **Read:** sections 14.1; also 12.6 to the bottom of p. 837[3]; and 14.2 (skip the theory, focus on examples 1–3, the definition of continuity, examples 7–8).
- **Work:** $\underline{\text{Problem 1}}$ below.

  14.1: (13), (25), (32), $\underline{36}$, (41), (47), $\underline{61}$, (63), $\underline{68}$, (72), (80).[4]

  14.2: (4), (7), $\underline{9}$[5], $\underline{13}$, (27), (33), $\underline{39}$, (43).

Friday 2/12 – Partial derivatives, tangent plane, linear approximation

- **Read:** section 14.3 to bottom of p. 921.[6]
- **Work:** 14.3: (5), $\underline{10}$, (11), $\underline{24}$, (37), (41), (42), $\underline{47}$, (51), (53), (61), $\underline{69}$, $\underline{77}$, $\underline{79}$, $\underline{88}$, $\underline{97}$.[7]

Problem 1.

a) Sketch the graph of the function $f(x, y) = 3 - x^2 - y^2$.

b) Sketch the graph of the function $g(x, y) = \sqrt{x^2 + y^2}$.

c) Make a rough sketch of a contour plot for the function whose graph is depicted in Figure 10(a) on page 892 [6th ed: p. 860, 7th ed: p. 882].

d) Draw a contour plot of the function $f(x, y) = e^{y/x}$ showing several level curves.

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[1] **6th ed:** 13.3 p. 830-831; 13.4 to p. 841. **7th ed:** 13.3 to middle of p. 855; 13.4 to p. 865.
[2] **6th ed:** 13.2: (39), (45), $\underline{47}$, $\underline{49}$, $\underline{50}$. **7th ed:** 13.2: (41), (47), $\underline{51}$, $\underline{53}$, $\underline{54}$.
[3] **6th ed:** bottom of p. 808; **7th ed:** bottom of p. 830
[4] **6th ed:** 14.1: (11), (23), (30), $\underline{32}$, (35), (41), $\underline{55}$, (57), $\underline{62}$, (66), (74).
7th ed: 14.1: (13), (25), (32), $\underline{36}$, (39), (45), $\underline{59}$, (61), $\underline{66}$, (70), (78).
[5] **6th ed:** do the 7th/8th ed problem: $\lim_{(x,y)\to(0,0)}(x^4 - 4y^2)/(x^2 + 2y^2)$.
[6] **6th ed:** bottom of p. 886. **7th ed:** bottom of p. 909.
[7] **6th ed:** 14.3: (5), $\underline{10}$, (11), $\underline{24}$, (35), (39), (40), $\underline{45}$* see below, (49), (51), (59), $\underline{67}$, $\underline{73}$, $\underline{75}$, $\underline{82}$, $\underline{87}$.
*for 14.3 # 45: do instead 14.3 # 47 of 7th/8th ed.: $x^2 + 2y^2 + 3z^2 = 1$.
7th ed: 14.3: (5), $\underline{10}$, (11), $\underline{24}$, (37), (41), (42), $\underline{47}$, (51), (53), (61), $\underline{69}$, $\underline{77}$, $\underline{79}$, $\underline{88}$, $\underline{93}$.

Problem 2 (extra credit, hard)

The purpose of this problem is to determine the shape of the road on which a square wheel rolls smoothly.

Suppose that the square wheel has sidelength 2, and starts with its axle at the origin and its sides parallel to the coordinate axes, touching the road at the point $(0, -1)$. The shape of the road is described by a parametric equation $\mathbf{r}(s) = x(s)\hat{\imath} + y(s)\hat{\jmath}$, using as parameter the arclength s along the road.

a) Given any shape of the road $(x(s), y(s))$, suppose that the square wheel rolls on it without slipping. Find parametric equations for the position $(x_1(s), y_1(s))$ of the axle when the contact point between the road and the wheel is at $(x(s), y(s))$. Express your formulas for $(x_1(s), y_1(s))$ in terms of s, $x = x(s)$, $y = y(s)$ and their derivatives.

b) Now impose the condition that the axle moves on a horizontal trajectory. Compute the velocity of the axle in terms of $\mathbf{r}' = d\mathbf{r}/ds$ and $\mathbf{r}'' = d^2\mathbf{r}/ds^2$, and find a formula for s in terms of $x'' = d^2x/ds^2$ and $y'' = d^2y/ds^2$.

c) Use the fact that s measures arclength to show that $\mathbf{r}'$ and $\mathbf{r}''$ are perpendicular. Deduce from this and from (b) a formula for s in terms of dx/ds and dy/ds only.

d) We now wish to describe the road as the graph of a function $y = f(x)$. Give an integral formula expressing the arclength s as a function of x, and use the result of (c) to show that $g(x) = f'(x)$ is the solution of a differential equation.

e) Find a solution to the differential equation of (d) with $g(0) = 0$ (hint: try hyperbolic trigonometric functions); deduce the shape of the road $y = f(x)$. For which values of x is your formula valid? (what happens afterwards?)