Math 53 Homework 10

Due Wednesday 4/6/16 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 3/28: Triple integrals in spherical coordinates

- **Read:** section 15.8. [7th edition: 15.9.]
- **Work:** 15.8: (5), (7), (9), (13), 14, 15, (17), 19*, (23), 26[1], (29), 30, 33, 35, (39).[2]

  Problem 1 below.

* For 15.8 # 19: set up the integral *both* in cylindrical and in spherical coordinates.

Wednesday 3/30: Vector fields

- **Read:** section 16.1.
- **Work:** 16.1: (5), 11, 13, (15), 18, (21), 23*, (26), 31.[3]

* For 16.1 # 23: also describe geometrically the gradient vector field.

Friday 4/1: Line integrals

- **Read:** section 16.2.
- **Work:** 16.2: 1[4], 3, (7), (11), (15), 17, 20[5], (29), 32*, (39), (41), 42.

  Problem 2 below.

* For 16.2 # 32: for part (b), try to find a geometric argument instead! What is the direction of $\vec{F}$? Observe: $\vec{F} = x(x\hat{\imath} + y\hat{\jmath})$.

Problem 1. Recall that the *average value* of $f(x, y, z)$ over a region D in space is $\frac{1}{V(D)} \iiint_D f(x, y, z)\, dV$, $\quad V(D) =$ volume of D.

Set up the integral *both* in cylindrical and spherical coordinates for the average distance from a point in the solid sphere of radius a to a point on the surface, and evaluate both integrals. Put the point on the surface at the origin and make it the South pole of the sphere.

Problem 2.

Consider the vector field $\vec{F} = (x^2 y + \frac{1}{3}y^3)\,\hat{\imath}$, and let C be the portion of the graph $y = f(x)$ running from $(x_1, f(x_1))$ to $(x_2, f(x_2))$ (assume that $x_1 < x_2$, and f takes positive values). Show that the line integral $\int_C \vec{F} \cdot d\vec{r}$ is equal to the polar moment of inertia of the region R lying below C and above the x-axis (with density $\rho = 1$).

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[1] **6th/7th ed:** do the 8th ed problem: $\iiint_E \sqrt{x^2 + y^2 + z^2}\, dV$, where E lies above the cone $z = \sqrt{x^2 + y^2}$ and between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

[2] **7th ed:** same problem numbers but in 15.9 instead of 15.8.

[3] **6th ed:** 16.1 # 11 and 13 are different from the 7th/8th edition. Do either version.

[4] **6th/7th ed:** do the 8th ed problem: $\int_C y\, ds$, $C : x = t^2$, $y = 2t$, $0 \le t \le 3$.

[5] **6th/7th ed:** do 8th ed: $\vec{F}(x, y, z) = (x + y^2)\hat{\imath} + xz\hat{\jmath} + (y + z)\hat{\mathrm{k}}$, $\vec{r}(t) = t^2\hat{\imath} + t^3\hat{\jmath} - 2t\hat{\mathrm{k}}$, $0 \le t \le 2$.