

Math 53 – Practice Midterm 1 B – 75 minutes

Problem 1.

Let P , Q and R be the points at 1 on the x -axis, 2 on the y -axis and 3 on the z -axis, respectively.

- (4) Express \overrightarrow{QP} and \overrightarrow{QR} in terms of \hat{i} , \hat{j} and \hat{k} .
- (4) Find the cosine of the angle PQR .

Problem 2. Let $P = (1, 1, 1)$, $Q = (0, 3, 1)$ and $R = (0, 1, 4)$.

- (6) Find the area of the triangle PQR .
- (3) Find the plane through P , Q and R , expressed in the form $ax + by + cz = d$.
- (3) Is the line through $(1, 2, 3)$ and $(2, 2, 0)$ parallel to the plane in part (b)? Explain why or why not.

Problem 3. A ladybug is climbing on a Volkswagen Bug (= VW). In its starting position, the surface of the VW is represented by the unit semicircle $x^2 + y^2 = 1$, $y \geq 0$ in the xy -plane. The road is represented as the x -axis. At time $t = 0$ the ladybug starts at the front bumper, $(1, 0)$, and walks counterclockwise around the VW at unit speed relative to the VW. At the same time the VW moves to the right at speed 10.

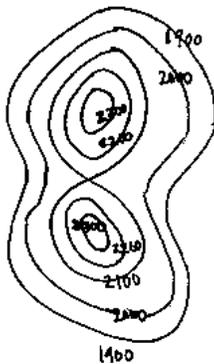
- (7) Find the parametric formula for the trajectory of the ladybug, and find its position when it reaches the rear bumper. (At $t = 0$, the rear bumper is at $(-1, 0)$.)
- (5) Compute the speed of the bug, and find where it is largest and smallest. Hint: It is easier to work with the square of the speed.

Problem 4.

(a) (4) Let $P(t)$ be a point with position vector $\vec{r}(t)$. Express the property that $P(t)$ lies on the plane $4x - 3y - 2z = 6$ in vector notation as an equation involving \vec{r} and the normal vector to the plane.

(b) (4) By differentiating your answer to (a), show that $\frac{d\vec{r}}{dt}$ is perpendicular to the normal vector to the plane.

Problem 5. (5) On the contour plot below, mark the portion(s) of the level curve $f = 2000$ on which $\frac{\partial f}{\partial y} \geq 0$.



Problem 6. Let $f(x, y) = x^2y^2 - x$.

- a) (4) Find ∇f at $(2, 1)$
- b) (2) Write the equation for the tangent plane to the graph of f at $(2, 1, 2)$.
- c) (2) Use a linear approximation to find the approximate value of $f(1.9, 1.1)$.
- d) (2) Find the directional derivative of f at $(2, 1)$ in the direction of $-\hat{i} + \hat{j}$.

Problem 7. a) (7) Find the critical points of

$$w = -3x^2 - 4xy - y^2 - 12y + 16x$$

and say what type each critical point is.

b) (8) Find the point of the first quadrant $x \geq 0$, $y \geq 0$ at which w is largest. Justify your answer.

Problem 8. Let $u = y/x$, $v = x^2 + y^2$, $w = w(u, v)$.

- a) (5) Express the partial derivatives w_x and w_y in terms of w_u and w_v (and x and y).
- b) (3) Express $xw_x + yw_y$ in terms of w_u and w_v . Write the coefficients as functions of u and v .
- c) (2) Find $xw_x + yw_y$ in case $w = v^5$.

Problem 9. a) (7) Find the Lagrange multiplier equations for the point of the surface

$$x^6 + y^6 + z^6 + xy + yz + zx = 6$$

at which x is largest. (Do not solve.)

b) (3) Given that x is largest at the point (x_0, y_0, z_0) , find the equation for the tangent plane to the surface at that point.

Problem 10. Suppose that $x^2 + y^3 - z^4 = 1$ and $z^3 + zx + xy = 3$.

- a) (5) Take the total differential of each of these equations.
- b) (5) The two surfaces in part (a) intersect in a curve along which y is a function of x . Find dy/dx at $(x, y, z) = (1, 1, 1)$.