The vector \( \vec{v} = \langle 1, 2, -1 \rangle \) and the plane \( x + y + 3z = 5 \) are

1. parallel
2. perpendicular
3. neither

\( \vec{v} \) is not \( \parallel \vec{N} \)

\[ \vec{v} \cdot \vec{N} = \langle 1, 2, -1 \rangle \cdot \langle 1, 1, 3 \rangle = 1 + 2 - 3 = 0 \]

\( \vec{v} \perp \vec{N} \)
A LINE AND A PLANE:

→ The line can be contained in the plane

→ or parallel to it

→ or intersects the plane in a point
This has to do with: 3x3 linear systems!

\[
\begin{align*}
  x + y + 2z &= 7 \quad (S_1) \\
  2x + y - z &= 4 \quad (S_2) \quad \text{3 planes: where do they intersect?} \\
  x + 2y + 3z &= 3 \quad (S_3)
\end{align*}
\]

How many solutions?

If the first 2 planes are not parallel, they intersect in a **line**.
Line and $P_3$ intersect in a point (1 solution)
Line parallel to $P_3$
(no solution)

Line contained in $P_3$
(infinite solutions)
RELATIVE POSITIONS OF 2 LINES:

→ parallel \quad \{ \text{in both cases, a unique plane containing both lines} \}

→ intersecting

→ skew lines = neither parallel nor intersecting

Then they lie on parallel planes

Distance between the 2 planes = shortest distance between points on the skew lines.
CYCLOID

\[ \overrightarrow{OC} = \langle a\theta, a \rangle \]
\[ \overrightarrow{CP} = \langle -a \sin \theta, -a \cos \theta \rangle \]
\[ \vec{r}(\theta) = \overrightarrow{OP} = \overrightarrow{OC} + \overrightarrow{CP} = \langle a\theta - a \sin \theta, a - a \cos \theta \rangle \]

\[ \text{distance on ground} = \text{arc length on wheel} \]