Math 53 Homework 7

Due Wednesday 10/11/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 10/2 – Midterm 1.
The problem below completes the material from Chapter 14, and can be attempted at any time, but is not directly related to the material on Midterm 1.

• Work: Problem 1 below.

Wednesday 10/4 – Polar coordinates.

• Read: section 10.3 through middle of p.663; 10.4 through top of p.671.¹
• Work: Problem 2 below.
  10.3: (11), (17), (19), (29), (37), (49), 51, (73).
  10.4: (5), 7, 29², (31), 35.

Friday 10/6 – Double integrals

• Read: sections 15.1, 15.2 (7th ed: 15.1, 15.2, 15.3).
• Work: 15.1: (21), 24, 29, 32, (36), (37), (43).

Problem 3 below.
  15.2: (1), (7), (17), (25), 27, (39), (46), (48), 51, 53, (61), 64.⁴

Problem 1. – Least-squares interpolation.

In experimental sciences, statistics, and many other fields, one often wishes to establish a linear relationship between two quantities (say \(x\) and \(y\)). Repeated experiments (or sampling of \(x\) and \(y\) for various individuals among the general population) give a set of experimental data \((x_1, y_1), \ldots, (x_n, y_n)\) (each pertaining to a different experiment or to a different individual). One then attempts to find the straight line \(y = mx + b\) that best fits the given experimental data. Least-squares interpolation is the most common method for doing so. (Note: the goal is to find \(m\) and \(b\), which describe the relation between \(x\) and \(y\) – we are not trying to solve for \(x\) and \(y\)!) We will first work out the general formula (following a problem in the book), then apply it in an example.

¹⁷th ed: 10.3 through middle of p.659; 10.4 through top of p.667.
²⁷th ed: do the 8th ed problem: \(r = 4\sin\theta\), \(r = 2\).
³¹⁷th ed: 15.2: (9), 12, 17, 20, (24), (25), (31).
⁴¹⁷th ed: 15.3: (1), (7), (17), (25), 27*, (37), (44), (46), 49, 51**, (59), 62.
* do the 8th edition problem: \(2x + y + z = 4\).
** do the 8th edition problem: \(\int_0^{1} \int_{\sqrt{y}}^{\sqrt{y^2+1}} dy \ dx\).
a) Do section 14.7 exercise #59 (7th ed: 14.7 #55). You don’t need to prove that the critical point is a minimum.

**Hint:** In this problem, the total square deviation \( \sum_{i=1}^{n} d_i^2 \) is a function of the two variables \( m \) and \( b \); all the \( x_i \) and \( y_i \) are constants (as part of the given experimental data). So this is a min-max problem in two variables, just like those we have seen in class. Looking for a critical point should give you the two equations. Proving that the critical point is indeed a minimum can be done (using the second derivative test or other methods) but takes quite a bit of effort; doing it is strictly optional.

b) The table below summarizes recent data for the yearly average CO\(_2\) content of the atmosphere in parts per million (ppm). (Source: NOAA ESRL).

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<tbody>
<tr>
<td>CO(_2) (ppm)</td>
<td>369.55</td>
<td>373.28</td>
<td>377.52</td>
<td>381.90</td>
<td>385.60</td>
<td>389.90</td>
<td>393.85</td>
<td>398.65</td>
<td>404.21</td>
</tr>
</tbody>
</table>

We take \( x \) to be the year since 2000, and \( y \) to be the CO\(_2\) concentration in ppm. So \((x_1, y_1) = (0, 369.55), \ldots, (x_9, y_9) = (16, 404.21)\).

Use the result of part (a) to write down the equations satisfied by the slope \( m \) and the intercept \( b \) of the best-fit line for this data. Next, solve these equations and give the values of \( m \) and \( b \). (Use a calculator or a computer!)

c) Compare the predicted values \( y = mx + b \) with the actual data for 2000, 2008 and 2016. How good is the fit? According to the best-fit line, how much CO\(_2\) will there be in the atmosphere in 2100?

Optional: produce a plot that shows the given data points and the best fit line.

**Problem 2.** For each of the given curves, find a Cartesian equation for it, and sketch it.

a) \( r = 3 \sin \theta \). b) \( r = 5 \sec \theta \). c) \( \theta = -\pi/3 \). d) \( r^2 \sin 2\theta = 2 \).

**Problem 3.** Evaluate the following double integrals:

a) \( \int_0^1 \int_{x^2}^x (1 + 2y) \, dy \, dx \),

b) \( \int_D \frac{y}{x^5 + 1} \, dA \), \( D = \{(x, y) \mid 0 \leq x \leq 1, \, 0 \leq y \leq x^2\} \),

c) \( \int_D 2xy \, dA \), \( D \) is the triangular region with vertices \((0,0)\), \((1,2)\), and \((0,3)\).