Math 53 Homework 5
Due Wednesday 9/27/17 in section
(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 9/18 – Chain rule
• Read: section 14.5.
• Work: 14.5: (1), 5, 71, (13), 15, (17), 18, (23), (39), 43, 45, (47), 51, (53).

Wednesday 9/20 – Gradient, directional derivatives
• Read: section 14.6.
• Work: 14.6: 1, (7), 9, (11), (21), 27, (37), 38, 41, (43), 49, (51), 56.

Friday 9/22 – Partial differential equations; max-min problems
• Work: 14.3: 75, (76), (80), 81
14.7: (1), 3, (7), 11, (23), (39), 43, (47), 49.

Problem 1. a) We consider again the contour plot of the function $f(x, y)$ depicted in HW 4 Problem 2. Imagine that we move from the point $(\frac{3}{2}, \frac{1}{2})$ along a straight line whose direction is given by a unit vector $(a, b)$, so $x(t) = \frac{3}{2} + at$ and $y(t) = \frac{1}{2} + bt$, and consider the value of $f(x(t), y(t))$ as a function of $t$.

(i) What are the possible direction(s) $(a, b)$ for which $\frac{df}{dt} = 0$ at $t = 0$? (Either sketch a picture showing a portion of level curve and the vector $(a, b)$, or give approximate angle from the x-axis). For each of these directions, when travelling along the chosen line, does the value of $f$ pass through a minimum or a minimum at $t = 0$?

(ii) What are the direction(s) $(a, b)$ for which $\frac{df}{dt}$ at $t = 0$ is largest, resp. smallest? Measure the contour plot to estimate the values of $\frac{df}{dt}$ at $t = 0$ for those directions.

b) Recall that the contour plot depicted in HW 4 Problem 2 corresponds to $f(x, y) = x(x - 1)(x - 2) + (y - 1)(x - y)$. At $(\frac{3}{2}, \frac{1}{2})$:

(i) find the unit vectors $(a, b)$ such that the derivative of $f$ along the line $x(t) = \frac{3}{2} + at$, $y(t) = \frac{1}{2} + bt$ is zero at $t = 0$.

(ii) find the unit vectors $(a, b)$ such that the derivative of $f$ along the line $x(t) = \frac{3}{2} + at$, $y(t) = \frac{1}{2} + bt$ is largest, resp. smallest at $t = 0$, and calculate the value of $\frac{df}{dt}$ at $t = 0$ for those directions.

17th ed: do the 8th ed version: $z = (x - y)^5$, $x = s^2 t$, $y = s t^2$.
27th ed: do the 8th ed version: $w = f(x, y, z)$, where $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$.
37th ed: do the 8th ed version: show that $c(x, t) = (4\piDt)^{-1/2}e^{-x^2/(4Dt)}$ is a solution of the diffusion equation $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$.
47th ed: 14.7: (1), 3, (7), 11 with $f(x, y) = x^3 - 3x + 3xy^2$, (21), (37), 41, (45), 47.
Problem 2.
Find the maximum rate of change of \( f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \) at the point \((3, 6, -2)\), and the direction in which it occurs. Optional: find a geometric interpretation to your answer.

Problem 3.

a) Find the direction from \((4, 2, 3)\) in which \( g(x, y, z) = x^2 + y^2 - 6z \) decreases fastest.

b) Follow the line in the direction you found in part (a) to estimate, using linear approximation, the location of the point closest to \((4, 2, 3)\) at which \( g = 0 \). Do not use a calculator. Express your answer using fractions. Next, use a calculator to evaluate \( g \) at your point. (The value should be reasonably small.)

(Hint: in the linear approximation, the closest point where \( g = 0 \) lies on the line through \((4, 2, 3)\) in the direction found in (a). Find a parametric equation for this line, then use linear approximation to estimate the value of \( g \) at a point on the line.)