Math 53 Homework 4
Due Wednesday 9/20/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 9/11 – Functions of several variables
• Read: sections 14.1; also 12.6 to the bottom of p. 8371; and 14.2 (skip the theory, focus on examples 1–3, the definition of continuity, examples 7–8).
• Work: Problem 1 below.
  14.1: (13), (25), (32), 36, (41), (47), 61, (63), 68, (72), (80).2
  14.2: (4), (7), 9, 13, (27), (33), 39, (43).

Wednesday 9/13 – Partial derivatives, tangent plane, linear approximation
• Read: section 14.3 to bottom of p. 921.3
• Work: 14.3: (5), 10, (11), 24, (37), (41), (42), 47, (51), (53), (61), 69, 77, 79, 88, 97.4

Problem 2 and 3 (next page)

Friday 9/15 – Tangent plane, linear approximation
• Read: section 14.4†.
  † Please don’t mix differentials like \(dz\) with numerical differences like \(\Delta x\) or \(\Delta y\). Statements such as “\(dx = \Delta x\)” are to be avoided. See lecture.
• Work: 14.4: (1), 5, 185, (19), 21, (25), 28, 331, 381, (39).
  Problem 4 (next page).
  † Whenever the book says “use differentials to estimate ...”, read “use linear approximation to estimate ...”.

Problem 1.

a) Sketch the graph of the function \(f(x, y) = 3 - x^2 - y^2\).
b) Sketch the graph of the function \(g(x, y) = \sqrt{x^2 + y^2}\).
c) Make a rough sketch of a contour plot for the function whose graph is depicted in Figure 10(a) on page 892 [7th ed: p. 882].
d) Draw a contour plot of the function \(f(x, y) = e^{y/x}\) showing several level curves.

Problem 2. The figure next page is the contour plot of a function of two variables \(f(x, y)\), for \(x\) and \(y\) ranging between 0 to 2 (scale: 1 unit = 5 cm; spacing between contour levels: 0.2).

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17th ed: bottom of p. 830
27th ed: 14.1: (13), (25), (32), 36, (39), (45), 59, (61), 66, (70), (78).
47th ed: 14.3: (5), 10, (11), 24, (37), (41), (42), 47, (51), (53), (61), 69, 77, 79, 88, 93.
57th ed: do the 8th ed version, \((y - 1)/(x + 1) \approx x + y - 1\).
a) Use the contour plot to determine whether $f_x$ and $f_y$ are $> 0$, $= 0$, or $< 0$ at the point $(\frac{3}{2}, \frac{1}{2})$. Same question at the point $(1, 1)$?

b) Use the contour plot to find two points where $f_x = f_y = 0$, and give their approximate coordinates. What happens to the level curves of $f$ through these points? For each of the two points, describe what happens when you move towards North, South, East, West: does the value of $f$ go up, down, or does it stay exactly the same?

Problem 3.
The function in the previous problem is $f(x, y) = x(x - 1)(x - 2) + (y - 1)(x - y)$.

a) Calculate the actual values of the partial derivatives at $(\frac{3}{2}, \frac{1}{2})$ and $(1, 1)$.

b) Find the points where $f_x = f_y = 0$, and calculate the second partial derivatives $f_{xx}$ and $f_{yy}$ at these points. Relate your answer to your findings in Problem 2(b).

Problem 4.
Assume that $b^2 - 4c > 0$ so that $x^2 + bx + c = 0$ has two roots. Let $r$ denote the larger root. Then $r$ is a function of $b$ and $c$.

a) Give an approximate formula for the small change $\Delta r$ in the value of $r$ produced by small changes $\Delta b$ and $\Delta c$ in the coefficients. Use this to calculate an approximate value for the larger root of $x^2 - 5.01x + 3.98 = 0$. Compare your answer with the exact value.

b) Starting from the equation $x^2 - 5x + 4 = 0$, is $r$ more sensitive to small changes in $b$ or $c$? (Justify your answer.)