Denis: Pair of pants HMS

wrapped Fuk cat

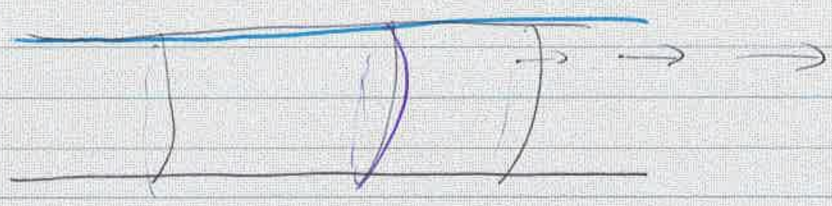
AAEKO 2013 ($n=3$)

Remember $\mathcal{W}(M)$ for M Liouville mfld
= exact sympl $(M, \omega = d\theta)$

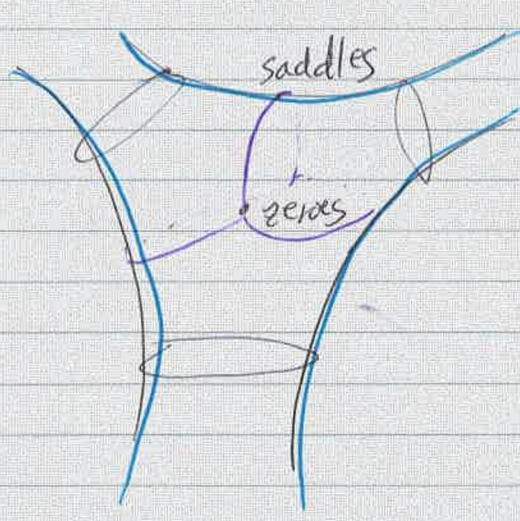
Z s.t. $\iota_Z \omega = \theta$ outwards pointing
& outside a compact set M^{in},

$$M = \underset{cpct}{M^{in}} \cup_{\partial M^{in}} \left(\underset{\alpha}{\partial M^{in}} \times [1, \infty) \right)$$

ω is $d(r\alpha)$ ↑ r here



Liouville vf



''

skeleta where it retracts, ascending mflds (Morse fn)

=

Lagrangian: exact $(\theta|_L = df)$ f cptly supp
conical at ∞

$$L = L^{in} \cup \left(\underset{\text{Legendrian}}{\Lambda} \times [1, \infty) \right)$$

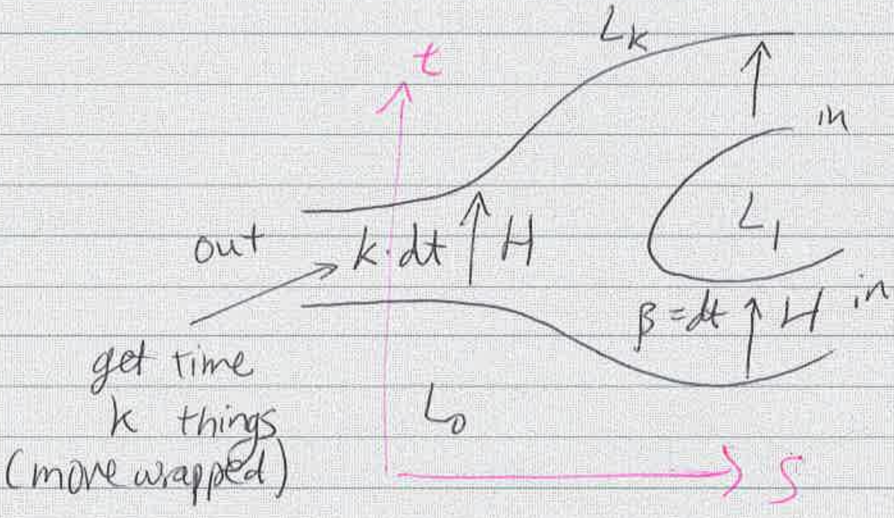
(Z tangent to L outside M^{in})

Ham. perturbation: $H(x,r) = r^2$ in the cyl ends

$CW(L_0, L_1)$ = gen by time 1 chords of X_H from L_0 to L_1
$X_H = 2r \cdot \text{Reeb}$

$\mu^k : CW(L_{k-1}, L_k) \otimes \cdots \otimes CW(L_0, L_1) \to CW(L_0, L_k)$

$k+1$ strip like ends e.g. μ^2

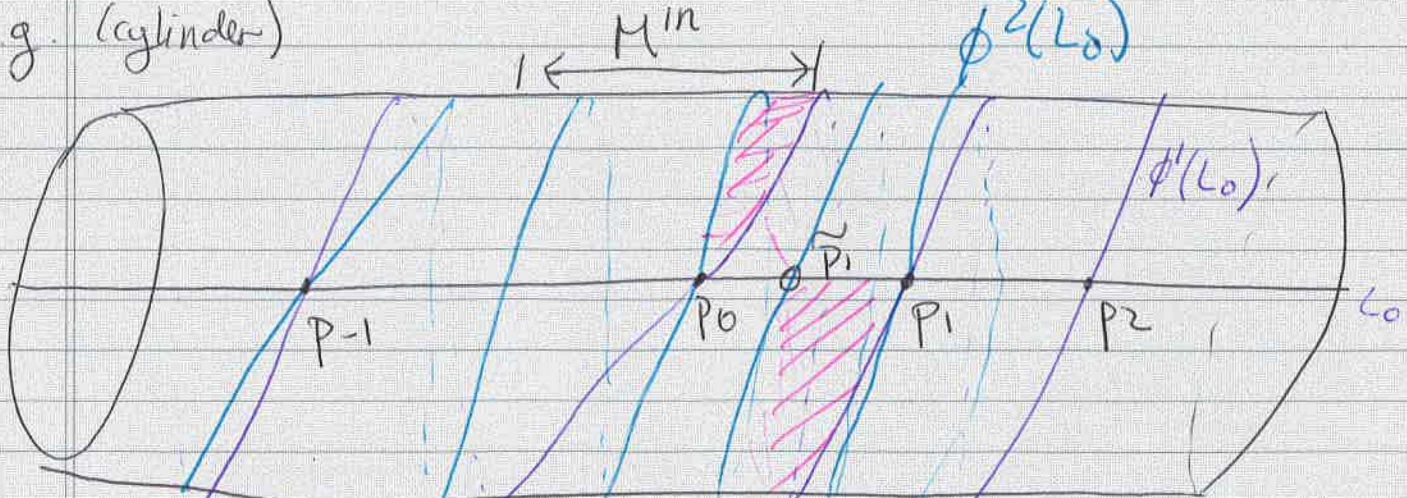


$(du - X_H \otimes \beta)^{0,1} = 0$

need $d\beta \le 0$ for cpctness, energy estimates

Solution: Abouzaid rescaling trick: rescale L_0, L_k

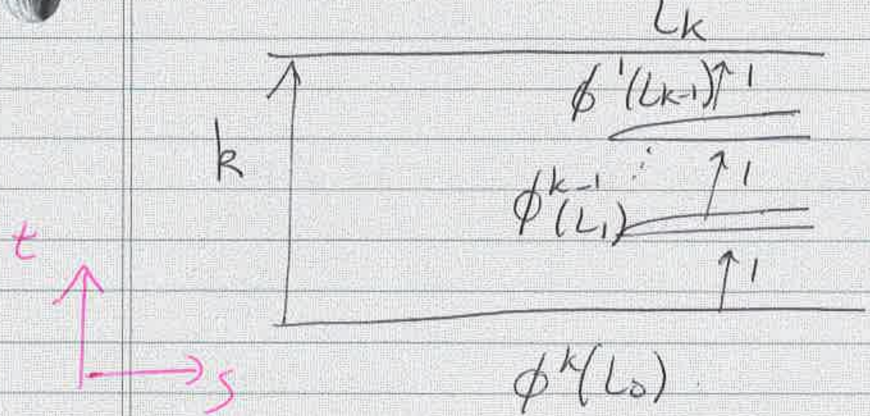
e.g. (cylinder)



time 1 chords $L_0 \to L_0 \overset{\text{same}}{\underset{\text{as}}{\Longleftrightarrow}} \phi_H^1(L_0) \cap L_0$

Rewrite trick Have u s.t. $(du - X_H \otimes \beta)^{0,1} = 0$

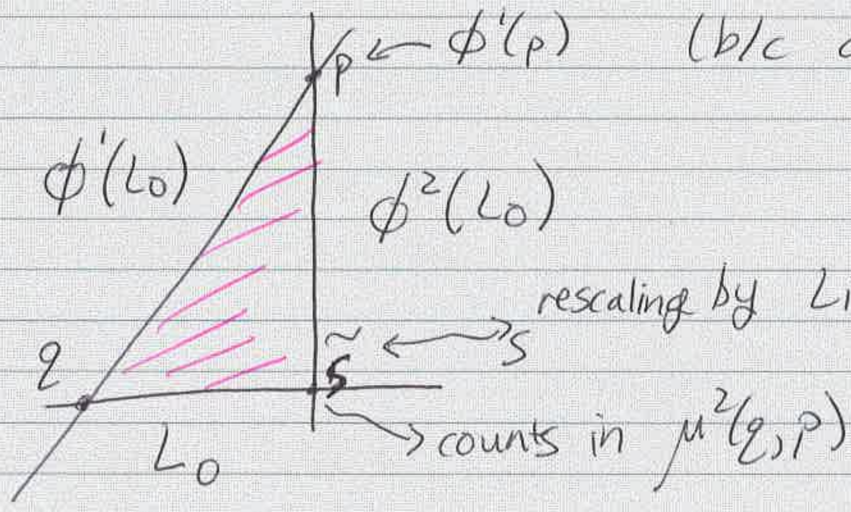
disk w/ slits



but change length of slits (a priori looks like one strip k times bigger than others)

Then $v(s,t) = \phi_H^{k-t}(u(s,t))$ solves $(dv)^{0,1}_{\phi^* J} = 0$

"Modify Lagr bdry condns"



(b/c applied ϕ^1 to everything)

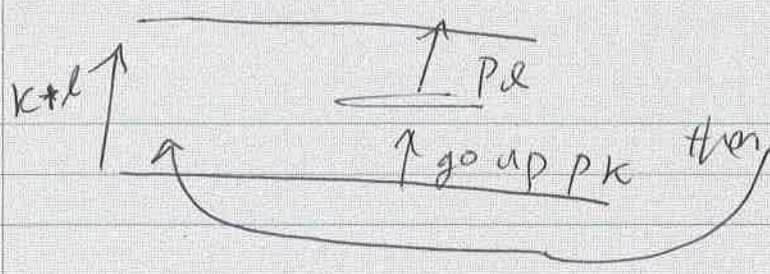
rescaling by Liouville flow $r \longleftrightarrow 2r$

$\to$ counts in $\mu^2(q,p)$

$\phi^1(L_0) \cap L_0 = \{ p_n \mid n \in \mathbb{Z} \}$

$\mu^2(p_1, p_0) = p_1$ (check it's the only one)

$\mu^2(p_k, p_\ell) = p_{k+\ell}$ $\to$ unroll to universal cover



gradings by homology classes : says $\boxed{\text{only one}}$
w/ $\mu^2(p_k, p_l) = p_{k+l}$

μ^1 trivial b/c no bigons on universal cover

homology on cylinder : bdry of disc must sum to 0 in homology

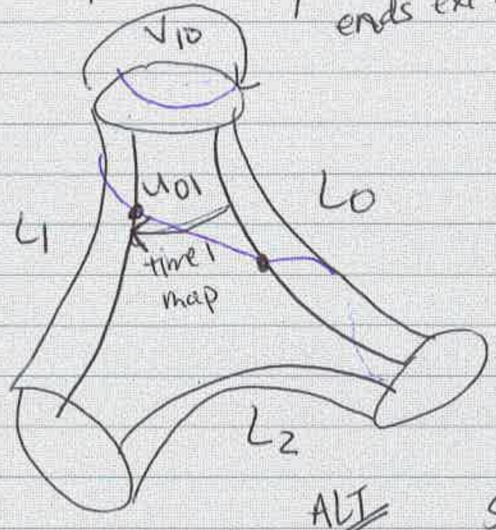
$HW^*(L_0, L_0) \simeq k[x^{\pm 1}]$

$p_k \longleftrightarrow x^k$

endos of str sheaves

& $\mu^{\geq 3} \equiv 0$ nts str sheaf generates derived cat on mirror.

e.g. $\boxed{\text{pair of pants}}$ ends extended @ ∞



Fact L_0, L_1, L_2 generate $W($$)$

(2 enough. they actually strongly generate → ~~[illegible]~~ don't need direct sums)

pf ~~[illegible]~~ Ab. generat^n criterion OR Lef thimbles (Seidel)

ALT skeleta  . . . (get all 3 too)

Goal A_∞-subcat w/ objs L_i

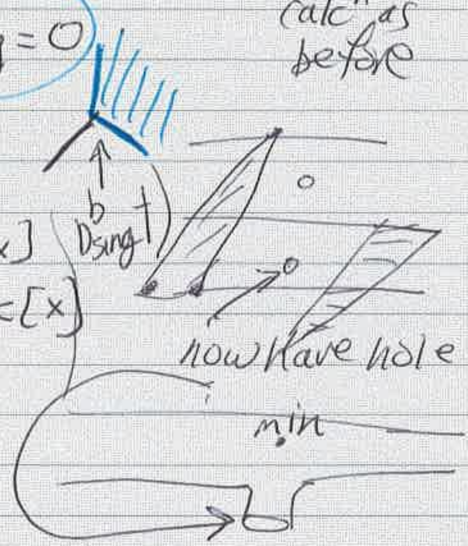
$\bigoplus_{i,j} CW(L_i, L_j)$ as A_∞-alg

$$\begin{tikzcd}
& L_0 \arrow[loop above, "\Bbbk[x,y]/xy=0"] \arrow[dl, bend right, "u_{01}\Bbbk[y]"'] \arrow[dr, bend left, "u_{20}\Bbbk[x]"] & \\
L_1 \arrow[loop left, "\Bbbk[y,z]/yz"] \arrow[ur, "v_{10}\Bbbk[y]"] \arrow[rr, bend right, "u_{12}\Bbbk[z]"'] & & L_2 \arrow[loop right, "\Bbbk[x,z]/xz"] \arrow[ul, "v_{02}\Bbbk[x]"'] \arrow[ll, bend right, "v_{21}\Bbbk[z]"']
\end{tikzcd}$$

$\boxed{u_{01}v_{10} = y}$

"go 1/2 way around"

similar calc^n as before



same y used even tho different things: b/c two ways to go around

$u \cdot \Bbbk[y] = \Bbbk[y] \cdot u$.

What about discs @ front & back? Yes but need μ^3 b/c all endpts are inputs output is Morse min:

$$\mu^3(u_{20}, u_{12}, u_{01}) = \pm 1_{L_0}$$

& 2 others

@ back: $\mu^3(v_{10}, v_{21}, v_{02}) = \pm 1_{L_0}$

& 2 others

interesting stuff happens @ sings

there's a μ^4 ..... don't want to keep computing

$\longrightarrow$ HH to the rescue

Hochschild cohomology calc^n:

$\rightsquigarrow$ Let A = algebra defined above (ie $\mu^1=0$, μ^2 given by prod str, $\mu^{\geq 3}=0$)

Q] Want to classify all A∞-algs $\mathcal{A}$ w/ $\mu^1=0$, $\mu^2=\mu^2_A$, $\mu^{k\geq 3}$?

compatible w/ all $\mathbb{Z}$-gradings st

deg u_{ij}, v_{ji} odd $\leftarrow$ b/c trivializing tangent NOT squared (oriented)

deg u_{01} + deg u_{12} + deg u_{20} = 1

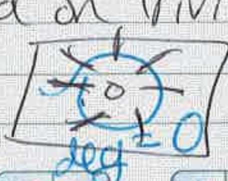
— " — with v — " —

(or: trivialize tangent bdle, get Maslov, look @ homology ...)

these determine A∞-str
$$\begin{cases} a = \text{coeff of } 1_{L_0} \text{ in } \mu^3(u_{20}, u_{12}, u_{20}) \\ b = \text{— '' —} \end{cases}$$

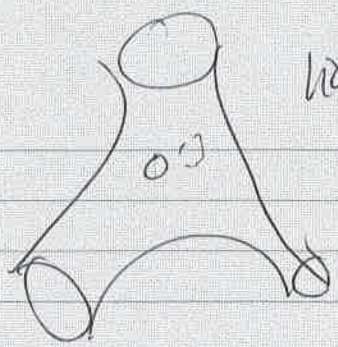
ie up to A∞-iso, all such A∞-strs classified by (a,b).

gradings depend on trivialization of tangent bdle:



or

are 2 different trivializations



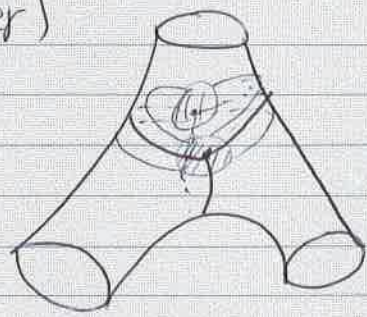
how to close?
how to know orbifold pt?

OC maps HH $\overset{\sim}{\leadsto}$ SH or QH

Metaprinciple: all algebra w/ Fuk cat has geometric realization

(Nadler)

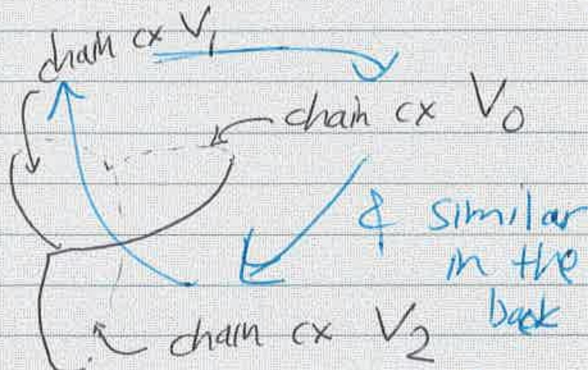
skeleta



e.g. Seidel Lagrangian
go 2× around graph

don't need wrapped b/c compact

Compact Lagrns $\longleftrightarrow$ constr. sheaves derived over skeleton w/ special rule @ 
Kontsevich



make cpct interact w/ noncpct

view as modules over wrapped Fuk cat

CF(L, ?)

triple, w/ 2 cones | cpt Lagrn is just this data

Mirror $(\mathbb{C}^3, W = +xyz)$ $\quad \mathbb{C}^3 = \operatorname{Spec} R$, $R = \mathbb{C}[x,y,z]$

mx factorizations ← will compare to wrapped Fuk cat

$MF(\mathbb{C}^3, W)$ (Eisenbud .... Orlov)

derived coh sheaf on $\mathbb{C}^3$: cx of free R-mods

complex $\quad R^{\oplus n_i} \xrightarrow{d_i} R^{\oplus n_j} \xrightarrow{d_{i+1}} R^{\oplus n_k}$

d_i mod 2

$d_i d_{i-1} = W \cdot id$

What $MF(\mathbb{C}^3, W)$ do we know?

1×1 mxes

$$\text{endos?}\left\{\begin{tikzcd} R \arrow[r, shift left, "(x)"] \arrow[d, "f"'] \arrow[dr, "k"] & R \arrow[l, shift left, "(yz)"] \arrow[d, "g"] \arrow[dl, "h"'] \\ R \arrow[r, shift left, "(x)"] & R \arrow[l, shift left, "(yz)"] \end{tikzcd}\right.$$

↓ even degree

odd degree

When is endo closed? nullhomotopic? (diagram commutes)

$f = g$ (polys in 3 vars)

get xh on both vertical ↓

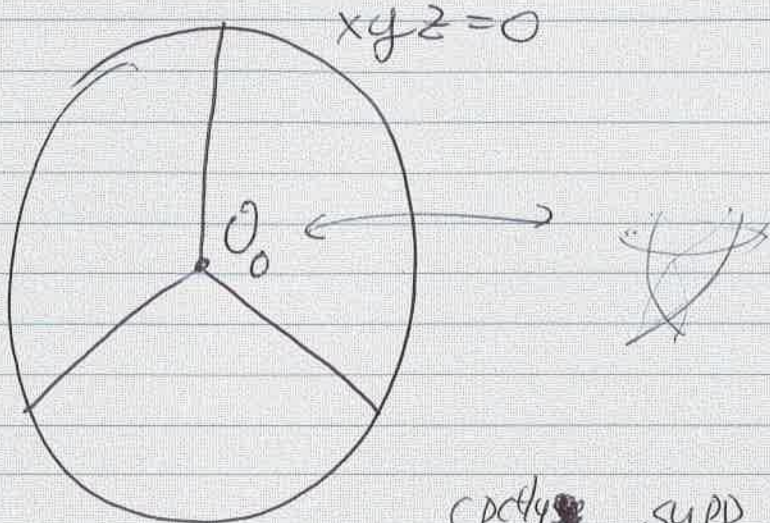
w/ k get yzk on both ends

Get $\mathrm{End}(\mathcal{E}_0) = \mathbb{C}[x,y,z]/(x, yz)$

$$= \mathbb{C}[y,z]/yz$$

$\boxed{\text{exact same as A-side!}}$

$xyz=0$

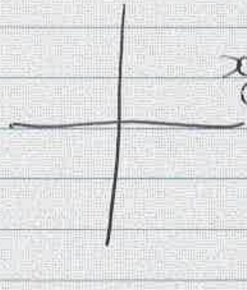


cpctly supp sheaves $\to$ just a pt

noncpct $\to$ wrapped Fuk cat

$$MF(W) \cong D^b_{sing}(W^{-1}(0)) = D^b Coh(W^{-1}(0))/Perf$$

e.g.



$xy=0$ $\quad \mathcal{O}_p \leftarrow \mathcal{O}_x \oplus \mathcal{O}_y \leftarrow \mathcal{O}$

$$\begin{tikzcd} \mathcal{O}_x \arrow[rr] & & \mathcal{O} \arrow[dl] \\ & \mathcal{O}_y \arrow[ul] & \end{tikzcd}$$