
1. (Milnor-Stasheff Problem 3-B) Given vector bundles \( F \subset E \), define the quotient bundle \( E/F \), and prove that it is locally trivial. If \( E \) has a Euclidean metric, show that \( F^\perp \simeq E/F \).

2. (Milnor-Stasheff Problem 3-E) Show that the set of isomorphism classes of rank 1 vector bundles over \( B \) forms an abelian group with respect to the tensor product operation. Show that a given real line bundle \( E \) possesses a Euclidean metric if and only if \( E \) represents an element of order \( \leq 2 \) in this group. [Note: for paracompact \( B \) this is always the case.]

3. Let \( A, B \) be paracompact spaces (feel free to assume they are CW-complexes if it helps), and let \( p : E \rightarrow B \) be a vector bundle over \( B \). Show that if two maps \( f_0, f_1 : A \rightarrow B \) are homotopic, then \( f_0^*E \) and \( f_1^*E \) are isomorphic vector bundles over \( A \).

(Corollary: every vector bundle over a contractible paracompact base \( B \) is trivial.)

**Hint:** There are several ways of doing this; here is one. Denoting by \( F \) the homotopy, show that the space of all linear isomorphisms between the fibers of \( f_0^*E \) and \( F^*E \) forms a locally trivial fiber bundle over \( A \times I \), and apply the homotopy lifting property to construct a section of this bundle.

4. (a) Show that the set of isomorphism classes of rank 1 Euclidean vector bundles over a CW-complex \( B \) has a natural bijection with the set of two-sheeted covering spaces of \( B \), and hence is isomorphic to \( H^1(B, \mathbb{Z}/2) \).

(b) Show that the isomorphism is given by the first Stiefel-Whitney class.

**Hint** for (b): recall that \( \mathbb{R}P^\infty = K(\mathbb{Z}/2; 1) \), and observe that the nontrivial element of \( H^1(\mathbb{R}P^\infty, \mathbb{Z}/2) \simeq \mathbb{Z}/2 \) is the first Stiefel-Whitney class of the universal line bundle.