Math 215B Spring 2012 – Homework 2 – due Wednesday Feb 29, 2012

Do Hatcher 4.2 problems 2, 22, 32, and EITHER 23 or 33.

Hatcher 4.2 Problem 2

Show the action of $\pi_1(\mathbb{RP}^n)$ on $\pi_n(\mathbb{RP}^n) \simeq \mathbb{Z}$ is trivial for n odd and nontrivial for n even.

Hatcher 4.2 Problem 22

Show that $H_{n+1}(K(G,n),\mathbb{Z}) = 0$ if $n > 1$. (Build a $K(G,n)$ from a Moore space $M(G,n)$ by attaching cells of dimension $> n+1$.)

(Recall: for $n > 1$, a Moore space $M(G,n)$ is a simply connected CW-complex such that $H_n(M(G,n)) = G$ and all other homology groups are zero.)

Hatcher 4.2 Problem 23

Extend the Hurewicz theorem by showing that if X is an $(n-1)$-connected CW complex, then the Hurewicz homomorphism $h : \pi_{n+1}(X) \to H_{n+1}(X)$ is surjective when $n > 1$, and when $n = 1$ show there is an isomorphism $H_2(X)/h(\pi_2(X)) \simeq H_2(K(\pi_1(X),1))$. [Build a $K(\pi_n(X),n)$ from X by attaching cells of dimension $n+2$ and greater, and then consider the homology sequence of the pair (Y,X) where Y is X with the $(n+2)$-cells of $K(\pi_n(X),n)$ attached. Note that the image of the boundary map $H_{n+2}(Y,X) \to H_{n+1}(X)$ coincides with the image of h, and $H_{n+1}(Y) \simeq H_{n+1}(K(\pi_n(X),n))$. The previous exercise is needed for the case $n > 1$.]

Hatcher 4.2 Problem 32

Show that if $S^k \to S^m \to S^n$ is a fiber bundle, then $k = n-1$ and $m = 2n-1$. (Look at the long exact sequence of homotopy groups.)

Hatcher 4.2 Problem 33

Show that if there were fiber bundles $S^{n-1} \to S^{2n-1} \to S^n$ for all n, then the groups $\pi_i(S^n)$ would be finitely generated free abelian groups, computable by induction, and nonzero whenever $i \geq n \geq 2$. [**Hint added:** observe that the inclusion of the fiber S^{n-1} into S^{2n-1} is nullhomotopic (why?)]