
Do Hatcher 4.2 problems 2, 22, 32, and EITHER 23 or 33.

Hatcher 4.2 Problem 2
Show the action of $\pi_1(\mathbb{RP}^n)$ on $\pi_n(\mathbb{RP}^n) \simeq \mathbb{Z}$ is trivial for $n$ odd and nontrivial for $n$ even.

Hatcher 4.2 Problem 22
Show that $H_{n+1}(K(G, n), \mathbb{Z}) = 0$ if $n > 1$. (Build a $K(G, n)$ from a Moore space $M(G, n)$ by attaching cells of dimension $> n + 1$.)

(Recall: for $n > 1$, a Moore space $M(G, n)$ is a simply connected CW-complex such that $H_n(M(G, n)) = G$ and all other homology groups are zero.)

Hatcher 4.2 Problem 23
Extend the Hurewicz theorem by showing that if $X$ is an $(n - 1)$-connected CW complex, then the Hurewicz homomorphism $h : \pi_n(X) \to H_{n+1}(X)$ is surjective when $n > 1$, and when $n = 1$ show there is an isomorphism $H_2(X)/h(\pi_2(X)) \simeq H_2(K(\pi_1(X), 1))$. [Build a $K(\pi_n(X), n)$ from $X$ by attaching cells of dimension $n + 2$ and greater, and then consider the homology sequence of the pair $(Y, X)$ where $Y$ is $X$ with the $(n+2)$-cells of $K(\pi_n(X), n)$ attached. Note that the image of the boundary map $H_{n+2}(Y, X) \to H_{n+1}(X)$ coincides with the image of $h$, and $H_{n+1}(Y) \simeq H_{n+1}(K(\pi_n(X), n))$. The previous exercise is needed for the case $n > 1$.]

Hatcher 4.2 Problem 32
Show that if $S^k \to S^m \to S^n$ is a fiber bundle, then $k = n - 1$ and $m = 2n - 1$. (Look at the long exact sequence of homotopy groups.)

Hatcher 4.2 Problem 33
Show that if there were fiber bundles $S^{n-1} \to S^{2n-1} \to S^n$ for all $n$, then the groups $\pi_i(S^n)$ would be finitely generated free abelian groups, computable by induction, and nonzero whenever $i \geq n \geq 2$. [Hint added: observe that the inclusion of the fiber $S^{n-1}$ into $S^{2n-1}$ is nullhomotopic (why?)]