ERRATA FOR
A SHORT COURSE ON SPECTRAL THEORY

William Arveson

Department of Mathematics
University of California
Berkeley CA 94720, USA
arveson@math.berkeley.edu

June, 2003

Abstract. This is a list of corrections I know about. Special thanks go to Launey Thomas for catching many corrections that I had missed. I welcome additional feedback from users of this text.

page 5. Exercise (5) is correctly stated, but the hint should be changed as follows. Hint: Apply Ascoli's theorem to KB_1.

page 11. In Exercise (1), the phrases $x_n \in X$ and $y \in X$ should be replaced with $x_n \in E$ and $y \in E$ respectively.

page 12. Theorem 1.4.2: the inequality (2) should be

$$\|e\|^{-1}\|x\| \le \|L_x\| \le c\|x\|.$$

page 13. In Exercise (2), E should be a Banach space.

page 14. Line 3: "...satisfies $\|\mathbf{1}\| = 1$".

page 19. Line 7: "...is so, fix $\lambda \in \sigma(x)$".

page 22. The lower formula displayed in line -12 should be

$$\inf_{z_1, z_2 \in I} \|(x + z_1)(y + z_2)\| \le \|\dot{x}\|\|\dot{y}\|.$$

page 28. Line 10 should read: "$K \subseteq D$ and $z_0 \notin D$."

page 32. Line 10: "that that" should be "that".

page 32. Line 22, $0 \ne \sigma_A(x)$ should be $0 \notin \sigma_A(x)$.

page 33. In Corollary 1, replace $x \in A$ with $x \in B$.

page 34. Line 20 should read: "as the norm $\|\mathcal{P}\|$ of the partition $\mathcal{P}$ tends to 0, where $\|\mathcal{P}\|$ is defined as the maximum of the arc lengths of the segments of C connecting γ_{k-1} to γ_k, $1 \le k \le n$". This definition of $\|\mathcal{P}\|$ implies that $\|\mathcal{P}\|$ must decrease as the partition $\mathcal{P}$ is refined, and simplifies Exercise 1 on p. 37.

page 44. In line -3, $\{p \in X : |f(p) - \lambda| < \epsilon\} = 0$ should be replaced with $\mu\{p \in X : |f(p) - \lambda| < \epsilon\} = 0$.

page 49. Line -5: "We do *not* assume continuity of θ."

page 51. Line 14: $sp(A)$.

page 56. Exercise (4) should end with the phrase: "...,and which commutes with the multiplication operator of Exercise (3)."

page 56. Exercise (5) should end as follows. Hint: Use the operator U of Exercise (4), together with the fact that $\mathbb{T} = X \cup Y$ where X is the top half and Y is the bottom half, to show that $L^2(\mathbb{T})$ can be identified with the direct sum

$$L^2(\mathbb{T}) \sim L^2(X, \mu) \oplus L^2(X, \mu),$$

and deduce that operators on this direct sum can be realized as 2×2 matrices over $\mathcal{B}(L^2(X, \mu))$. You should write down an *explicit* unitary operator that makes the above identification.

page 57. Exercise (7) is inappropriate for this section, since it requires the result of Theorem 4.1.2 which has not yet been proved. It should probably be moved to Section 4.1.

Replace "What is the commutant of B?" with "Compare the commutants of A and B".

page 59. Exercises (4) and (5) should be moved to page 63, as described in the next item. Reason: they require the fact that von Neumann algebras are generated by projections, which has not yet been proved.

page 63. Line 12: "σ-representation" should be hyphenated.

pp 63–64. Three more exercises should be added.

Exercise (3). a). Let X be a compact metric space, let $\pi : C(X) \to \mathcal{B}(H)$ be a nondegenerate representation of $C(X)$, and let $\tilde{\pi} : B(X) \to \mathcal{B}(H)$ be its extension to a σ-representation. Show that for every bounded Borel function $f \in B(X)$, the operator $\tilde{\pi}(f)$ belongs to the double commutant $\pi(C(X))''$ of the range of π.

b). Deduce that the von Neumann algebra $\pi(C(X))''$ is the norm-closed linear span of its projections.

Exercise (4). This is Exercise (4) from page 59. Hint: use the the functional calculus and the preceding exercise.

Exercise (5). Exercise (5) from page 59, verbatim.

page 67. Line 3 of Exercise (1) should read: "unitary operator $W : H_1 \to H_2$...".

page 72. In line 22, replace "see Exeercise (2) below" with "see Exercise (3) below".

page 78. Line -10: "unitilization" should be "unitalization".

page 91. In line 1, replace "Lemma 2.4.3" with "Lemma 3.2.7".

page 91. Line 11. Replace "is injective" with "is surjective".

page 98. Line -5: "$L - KB$" should be "$L + KB$".

page 103. Line -3 should read: "commutes with M_f".

page 104. In line -18, replace "the measure σ" with "the measure μ".

page 106. In line -12, replace "subace" with "subspace".

page 110. In the Exercises, the notation for Banach limits should be clarified as follows. Line 7 should read: "...we will write $\Lambda(a)$ or $\Lambda((a_n))$ for...". The displayed formula on line 12 should read

$$\langle \phi(A)\xi, \eta\rangle = \Lambda((\langle S^{*n}AS^n\xi, \eta\rangle)), \qquad \xi, \eta \in H^2.$$

page 110. Interchange f and g in the first paragraph of the proof of Theorem 4.3.1 as follows. "If $g \in C(\mathbb{T})$ and $f \in L^\infty$, ..., we may reduce to the case $g = \zeta^n$ and $f \in L^\infty$, $n \in \mathbb{Z}$."

pages 110–111. Line -2: (4.3.1) should be (4.6). Same change in Line 2 of page 111.

page 111. The displayed formula on line 7 needs a change in sign as follows

$$T_{f\zeta^n} - T_f T_{\zeta^n} = S^{*m}T_f - T_f S^{*m} = S^{*m}T_f(\mathbf{1} - S^m S^{*m}).$$

page 111. In line 20, replace "Theorem 4.3.1" with "Proposition 4.3.1".

page 116. In the displayed formula on line -4, replace ... $=$ ind$(T_{fg} + K)$ with ... $=$ ind$(T_{fg} - K)$.

page 117. In line 5, replace $\mathbb{Z}^+$ with $\mathbb{Z}_+$.
In line -13, replace $K = \ell^2(\mathbb{Z}^+) \subset H$ with $K = \ell^2(\mathbb{Z}_+) \subseteq H$.

page 118. Line -14 should read "...whose original proof was quite different."

page 119. Lines -8 to -7 should read "With $P_+ \in \mathcal{B}(L^2)$ denoting...".

page 120. Line 6: "$\bar{\phi}(z)$ vanishes..." should be "$\phi(z)$ vanishes...".

page 122. In line 11, "Lebeggue" should be "Lebesgue".

page 123. The displayed formula on line 15 should be

$$|\rho(x)|^2 = |\rho(\mathbf{1}^*x)|^2 \leq \rho(x^*x)\rho(\mathbf{1}).$$

page 124. Line 1 should read "...promotes naturally to a sesquilinear...".

page 124. The displayed equation in line 5 should end with

$$\ldots \implies x + N = 0 + N.$$

page 124. Line -6 should begin with $\pi(a^*) = \pi(a)^*$, $a \in A$.

page 125. In line -4, add a third part to Exercise (2) as follows:
(c) Deduce that any positive element $x \in A$ (i.e., x is self-adjoint and $\sigma(x) \subseteq [0, \infty)$) has a unique positive square root.

page 128. In lines 19 to 19, change "sub C^*-algebra" to "C^*-subalgebra".