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*Extremal marginal traces on $M_n \otimes M_n$ and quantum entanglement*

$M_n$ denotes the $C^*$-algebra of all $n \times n$ complex matrices. A *marginal trace* is a state $\rho$ on $M_n \otimes M_n$ with the property $\rho(a \otimes 1) = \rho(1 \otimes a) = \tau(a)$, $\tau$ being the tracial state of $M_n$.

In 2002, Parthasarathy showed that an extremal marginal trace on $M_2 \otimes M_2$ must be a pure (maximally entangled) state of $M_2 \otimes M_2$. Oliver Rudolf has since given an example of an extremal marginal trace of $M_3 \otimes M_3$ that is not pure; earlier, Landau and Streater gave a similar example on $M_4 \otimes M_4$. By analyzing the geometry of the compact space $K$ of marginal traces of rank $\leq 2$ on $M_n \otimes M_n$ for $n \geq 4$ in terms of completely positive maps, we show that the extremal ones are generic in the sense that they are an open dense subset of $K$, and moreover, that all extremals are maximally entangled. More explicitly, if you close your eyes and point to a marginal trace of rank at most 2, you will most likely have a maximally entangled state.

In this first lecture I will discuss basic issues, the so-called separability problem of quantum information theory, and give precise definitions of buzzwords like “maximally entangled states”.