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Aspects of supergroup gauge theory

Taro Kimura 木村太郎

Université Bourgogne Franche-Comté

Keywords

- Instanton counting
- Brane dynamics, Seiberg-Witten geometry
- Topological string, Geometric engineering
- Gauge/Bethe correspondence
- Intersecting defects = Supergroup gauge theory

References

- TK-Pestun: 1905.01513
- Chen-TK-Lee: 1908.04928, 2003.13514
- TK-Sugimoto: 2001.05735
- TK-Nieri: 2105.02776
- TK: 2012.11711 (Habilitation thesis)

Overview

Supergroup YM theory e.g. $G = U(n_+|n_-)$

$$\mathcal{L}_{YM} = -\frac{1}{2g^2}\,\mathrm{Str}\, F_{\mu\nu}F^{\mu\nu}$$

$$= -\frac{1}{2g^2}\,\mathrm{tr}_+ F\cdot F + \frac{1}{2g^2}\,\mathrm{tr}_- F\cdot F$$

wrong sign !

spectrum unbounded, non-unitary ...

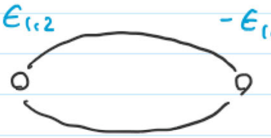
- Quiver realization $G \supset G_+ \times G_-$ $(U(n_+|n_-) \supset U(n_+)\times U(n_-))$

bifund.

$$\mathcal{A} = \left(\begin{array}{c|c} G_+ & G_+\times G_- \\ \hline G_+\times G_- & G_- \end{array}\right)$$

bifund.

(Ω-def.) $\epsilon_{1,2}$ $-\epsilon_{1,2}$ [DHJV]



G_+ G_- $(\hat{A}_1\text{-quiver})$

(cmplx.) τ $-\tau$

$(\text{“}\tau_+)$ $(\text{“}\tau_-)$

[physical cond. $\mathrm{Im}\,\tau > 0$]

ABJM : $U(n)_k \times U(m)_{-k}$ theory

- Brane realization

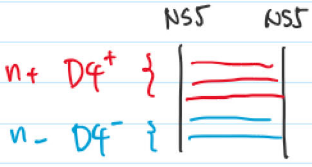
$U(n)$ gauge theory = stack of n branes

$U(n_+|n_-)$ theory = n_+ (positive) branes [OT]

& n_- (negative, ghost) branes

Rem : Negative brane $\neq$ anti-brane

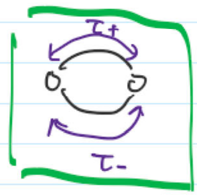
Hanany - Witten setup

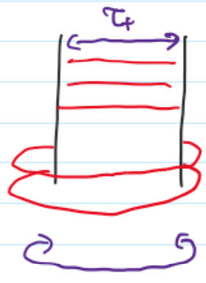


still BPS

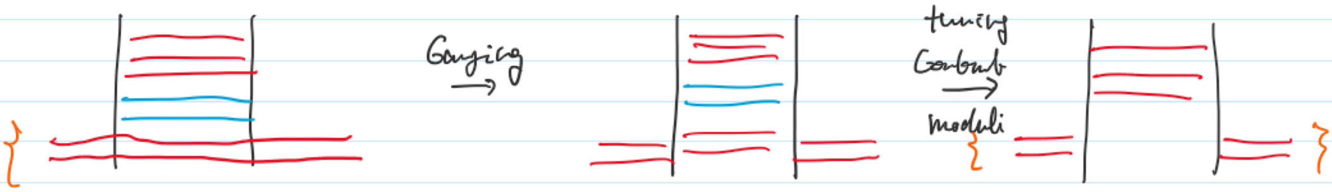
$\tau_- \to -\tau_+$

$\hat{A}_1$ quiver





decoupling $\tau_- \to \infty$

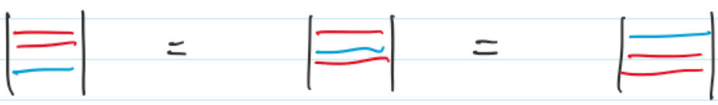


$SU(n_+)$ SQCD

w/ $2n_-$ flavors

Rem : Ordering is not unique

$SU(2|1)$



Dynkin : o—⊗ ⊗—⊗ ⊗—o

- Seiberg - Witten geometry [DHJV]

Pure $SU(n)$ SYM theory

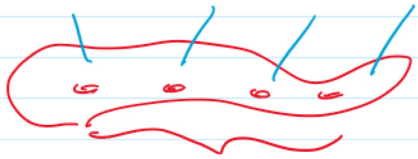
$$y + 1/y = \prod_{i=1}^{n} (x - a_i) = \det(x - \phi)$$

$$\boxed{y + \frac{1}{y} = \frac{\prod^{n_+}(x - a^+_i)}{\prod^{n_-}(x - a^-_i)}} = \operatorname{Sdet}(x - \phi)$$

$$\boxed{y + \frac{1}{y} = \frac{\prod^{n_+}(x-a^+_i)}{\prod^{n_-}(x-a^-_i)}} = \mathrm{Sdet}\,(x-\phi)$$

SU(n) SQCD

$$y + \frac{1}{y} = \frac{\prod_{i=1}^{n}(x-a_i)}{\sqrt{\prod_{f}^{n_f}(x-m_f)}}$$



n − 1

$$\left(n_f = 2n_- \; : \; m = (m_1, m_1, m_2, m_2, \cdots) \right)$$

$$\to \frac{\boxed{\prod_{i=1}^{n}(x-a_i)}}{\boxed{\prod_{f=1}^{n_-}(x-m_f)}} = \mathrm{Sdet}\,(x-\phi)$$

- Super instanton counting [KP]

ADHM construction = T dual

$(k_+|k_-)$ D0 $= k_+$ D0$^+$ ⊕ k_- $\overline{\text{D0}}$ $\quad (n_+|n_-)$ D4 $= n_+$ D4$^+$ ⊕ n_- D4$^-$

k - instanton in U(n) theory on T^4 (radius R) $\longrightarrow \mathbb{R}^4$

$\updownarrow$

$(n_+|n_-)$ n - instanton in $(k_+|k_-)$ U(k) theory on $\check{T}^4$ ($\check{R} = 1/R$) $\longrightarrow$ {pt}

Vector sp. : $N = \mathbb{C}^n \to \mathbb{C}^{n_+|n_-}$, $K = \mathbb{C}^k \to \mathbb{C}^{k_+|k_-}$

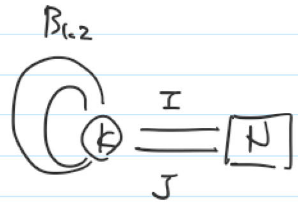
⇒ Chan - Paton for D0$^\pm$, D4$^\pm$

	instanton	anti - inst.	negative inst	anti - negative
SD or ASD	ASD	SD	ASD	SD
top #	+	−	−	+
D - brane	D0$^+$	$\overline{\text{D0}}^+$	D0$^-$	$\overline{\text{D0}}^-$

$$\begin{cases} \text{D0}^+ - \overline{\text{D0}}^+ : \text{non - BPS} & (\text{ASD} + \text{SD}) \\ \text{D0}^+ - \text{D0}^- : \text{BPS} & (\text{ASD}) \end{cases}$$

∘ ADHM data

$$\begin{cases} B_{1,2} : K \to K \\ I : N \to K \\ J : K \to N \end{cases}$$



∘ ADHM moduli sp.

$$\mathcal{M}_{n,k} = \mu^{-1}(0) /\!/\!/ U(k)$$

$$\Rightarrow \tilde{\mathcal{M}} = \mu_{\mathbb{C}}^{-1}(0) /\!/ GL(k) \quad \text{w/ stability cond.}$$

$$\mu_{\mathbb{R}} = 0 \quad \to \quad \mu_{\mathbb{R}} = \zeta_{\neq 0}$$

Stab. cond.

$$K = \begin{cases} \mathbb{C}[B_1, B_2]\, I(N) & (\zeta > 0) \\ \mathbb{C}[B_1^\dagger, B_2^\dagger]\, J^\dagger(N) & (\zeta < 0) \end{cases}$$

Super stab. cond.

$$\mu_{\mathbb{R}} = \begin{pmatrix} +\zeta & \\ & -\zeta \end{pmatrix}, \quad \mu_{\mathbb{C}} = 0$$

$$\begin{cases} K_+ = \mathbb{C}\,\underline{[B_1, B_2]}\, I(N_+) \\ K_- = \mathbb{C}\,\underline{[B_1^\dagger, B_2^\dagger]}\, J^\dagger(N_-) \end{cases}$$

Equiv. action : $U(1)^2 \subset SO(4)$

$$B_{1,2} \to q_{1,2} B_{1,2} \quad \text{w/ } q_{1,2} = e^{\epsilon_{1,2}}$$

$$\begin{cases} \text{positive sector} : (\underline{\epsilon_1, \epsilon_2}) \\ \text{negative} \quad " \quad : (\underline{-\epsilon_1, -\epsilon_2}) \end{cases}$$

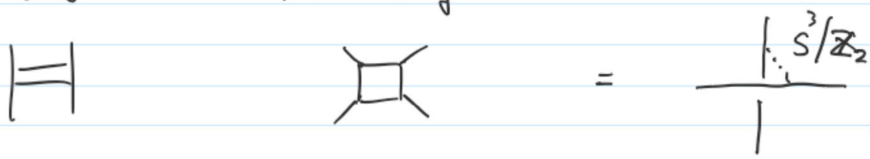
G - SYM	${}^L\hat{G}$ - Toda	G = Supergroup.
G - $N=2^*$	G - Calogero	
G - SQCD	$SL(2)$ XXX	w/ positive & negative magnons

G - SQCD $\quad \underbrace{SL(2)}_{\downarrow\, SL(n|m)}\ XXX$ w/ positive & negative magnons

- Topological string

Geometric engineering : A_n - singularity (ADE) $\Rightarrow$ $SU(n+1)$ theory

Brane web = toric diag



Taub - NUT : $TN_n \sim S^1 \times \mathbb{R}^3 \xrightarrow{S^1 \to \infty}$ ALE : $\mathbb{R}^4/\mathbb{Z}_n$

} n - centered TN

$$V(\vec{x}) = 1 + \sum_{i=1}^{n} \frac{+1}{|\vec{x} - \vec{x}_i|}$$

positive $\qquad$ negative

$$\to 1 + \sum_{i=1}^{n_+} \frac{+1}{|\vec{x} - \vec{x}_i^{+}|} + \sum_{i=1}^{n_-} \frac{-1}{|\vec{x} - \vec{x}_i^{-}|}$$

$$\xrightarrow{\text{ALE lim}} \mathbb{R}^4/\mathbb{Z}_{n_+ - n_-}$$

$SU(n_+|n_-)$ vs $SU(n_+ - n_-)$

cf. signature change

[DHJV]

Resolution

$$y + y^{-1} = x^n \to x^n + \cdots = \det(x - \phi) \qquad SU(n)$$

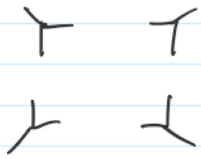
$$\to \frac{x^{n_+} + \cdots}{x^{n_-} + \cdots} = \mathrm{sdet}(x - \phi) \qquad SU(n_+|n_-)$$

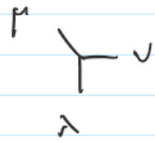
w/ $n_+ - n_- = n$

$\Rightarrow$ A_n - sing $\approx$ $A_{n_+|n_-}$ - sing. $\quad (n_+ - n_- = n)$.

$\Rightarrow$ A_n - sing $\approx$ $A_{n_+|n_-}$ - sing. $(n_+ - n_- = n)$.

∘ Topological vertex for supergroup theory [KS]

 $=$ 

where 

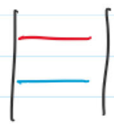
$= C_{\mu\lambda\nu}(q_1, q_2^{-1})$

cf. standard convention : $(q, t) = (q_1, q_2^{-1})$

∘ Negative node : $q_{1,2} \to q_{1,2}^{-1}$

$\Rightarrow$ Negative vertex (anti) : $\bar{C}_{\mu\lambda\nu}(q_1^{-1}, q_2) =$ 

e.g. $U(1|1)$ theory

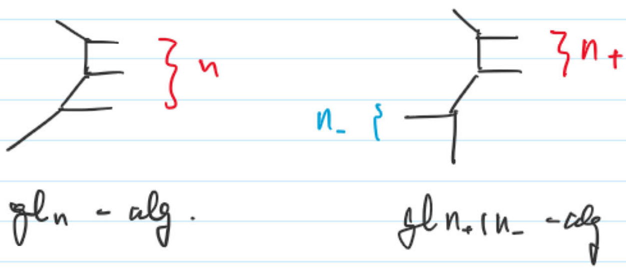
 $\Longleftrightarrow$ 

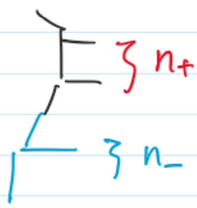
∘ Unrefined slice : $q_1 q_2 = 1$ $(q = t)$

 $=$  (∗ framing)

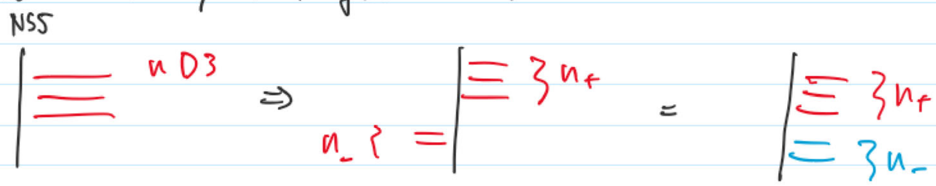
$C_{\dots}(q)$ $(\bar{C}(q) =)$ $C(q^{-1})$

Corner VOA

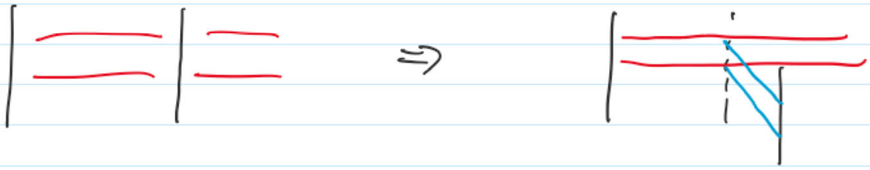


$=$ 

Gaiotto – Witten / Mikhaylov – Witten



- Intersecting defects & supergroup str. [KN]

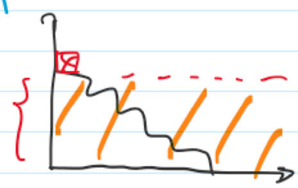


Ω - deformation :

$$a = m + n \cdot \epsilon_1 + m \cdot \epsilon_2$$

SU(n) rep.

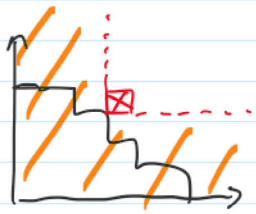
at most n rows



trapped inst.

= vortex

SU(n|m) rep.



- pit cond. [BEM]
- fat hook

intersecting defects.

$\mathbb{C}_{\epsilon_1} \cup \mathbb{C}_{\epsilon_2}$

- 5d / 3d (4d / 2d) correspondence cf. triality

(non-supergroup)

5d theory : $Z = \sum_{\lambda} Z_\lambda$

$\Downarrow$

3d theory (Higgs br.) : $Z = \sum_{\{p_i\}} Z_x$

$\Downarrow$

3d theory (Coulomb br.) : $Z = \oint \cdots$

Coulomb br. formula

$$Z_{n|m} = \oint d^n z \, d^m w \; \underbrace{\frac{\Delta(z; q, t) \, \Delta(w; t, q)}{\prod_{i,j} \left(1 - z_i / w_j\right)^2}}_{\text{super Macdonald measure}} \times \cdots$$

$$\underbrace{\prod_{i,j} \left(1 - \ldots / w_j\right)}_{\text{super Macdonald measure}}$$

$$\Delta(z; q, t) = \prod_{i \neq j}^{n} (z_i/z_j ; \underset{\mathbb{C}_q}{q}) \Big/ \left(\underset{\text{def mass}}{t} \frac{z_i}{z_j} ; q\right)$$