

Math-113, Homework 8, non-textbook problems

A. Let $A = \{a_1\}$ be a singleton set.

- Does there exist a group homomorphism $\phi : F[A] \rightarrow D_{10}$ from the free group generated by A to the Dihedral group with 10 elements (i.e. to a group of symmetries of a 5-gon)?
- Does there exist an injective group homomorphism $\phi : F[A] \rightarrow D_{10}$?
- Does there exist a surjective group homomorphism $\phi : F[A] \rightarrow D_{10}$?

In all cases justify your answer, i.e. give an example of a homomorphism or prove that it doesn't exist.

B. Let $B = \{b_1, b_2\}$ be a set with two elements. Prove that for all $n \in \mathbb{N}$ there exists a surjective homomorphism

$$\phi : F[B] \rightarrow D_{2n}.$$