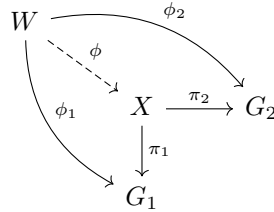


Math-113, Homework 7, non-textbook problems

A. Prove that the direct product $X = G_1 \times G_2$ of two groups satisfy the so-called *universal property*:

For every group W and pair of group homomorphisms $\phi_1 : W \rightarrow G_1$ and $\phi_2 : W \rightarrow G_2$ there exists a unique group homomorphism $\phi : W \rightarrow X$ s.t. $\phi_1 = \pi_1 \circ \phi$ and $\phi_2 = \pi_2 \circ \phi$. (1)



Hint: First construct a function ϕ and prove that it is unique. Then show that the function ϕ , which you have constructed satisfy homomorphism property.

B*. (optional, not for the grade) Prove that if there exists another group Y together with a pair of maps $\pi'_1 : Y \rightarrow G_1$ and $\pi'_2 : Y \rightarrow G_2$, s.t. the triple (Y, π'_1, π'_2) satisfy universal property (1), then $Y \simeq X$.

Remark. Note that the statement of the Problem B implies that the group X satisfying universal property (1) is unique up to isomorphism. This allows one to use (1) as an alternative definition of a direct product of groups. Of course, we will not use this approach in the current course, however, it is worth noting that such definition of a “product” can be used in many areas of mathematics well beyond the framework of groups and group homomorphisms.