Math-113, Homework 6, non-textbook problem

A. Let D_{2n} stand for the group of symmetries of a regular n-gon. Denote by $r \in D_{2n}$, the rotation by $\frac{2\pi}{n}$ counterclockwise. Also denote by $s \in D_{2n}$, the reflection about the axis passing through the center of n-gon and vertex 1. (In our case, reflection about the X-axis).

- Recall that every element $g \in D_{2n}$ is uniquely determined by where it sends the first edge $(1, 2)$. Prove that

$$D_{2n} = \left\{1, r, \ldots, r^{n-1}, s, rs, r^2 s, \ldots, r^{n-1}s\right\} \tag{1}$$

- Prove that $D_{2n} \leqslant S_n$ is a subgroup of a permutation group of n vertices of an n-gon.
- Describe the permutation of vertices corresponding to elements s and r and prove that

$$srs = r^{-1}. \tag{2}$$

- Let $g = r^{17}s \in D_{2020}$ and $h = r^{16}s \in D_{2020}$ be elements of the dihedral group of of order 2020. Use equation (2) to present the product $g \circ h$ in the form (1).