Math-113, Homework 3, non-textbook problems

A. Let S_3 be a permutation group on three letters.

- List all the elements of S_3 and compute their orders. Based on this calculation determine whether it is true for S_3 that the orders of elements divide the order of a group.
- Consider the subset

$$H = \{a \in G \,|\, a^2 = 1\}$$

  and prove that H is not a subgroup.
- Explain, why there is no contradiction with problem B from the previous homework.

B* (Optional, not graded). Again, let S_3 be a permutation group on three letters. For each $x \in S_3$, let $\lambda_x \in S_{S_3}$ denote the permutation on S_3 corresponding to the left multiplication by x. Enumerate all elements in S_3 in any order you would like. Because there are exactly six elements in S_3, now we can think of λ_x as an element of S_6.

- Write $\lambda_{\sigma_{12}}, \lambda_{\sigma_{23}}$ in the standard two row notation. (note that each row must be of length six).
- Determine the order of a permutation $\lambda_{\sigma_{12}} \in S_6$.
- Verify $\lambda_{\sigma_{12}}\lambda_{\sigma_{23}}\lambda_{\sigma_{12}} = \lambda_{\sigma_{23}}\lambda_{\sigma_{12}}\lambda_{\sigma_{23}}$.