Math-113, Homework 2, non-textbook problems

A. Consider the set

$$G := \{\begin{pmatrix}1 & a\\ 0 & 1\end{pmatrix} : a \in \mathbb{Z}\}.$$

- Show that it is a group, under matrix multiplication.
- Show that the map

$$\begin{aligned}(\mathbb{Z}, +) &\to G,\\ a &\mapsto \begin{pmatrix}1 & a\\ 0 & 1\end{pmatrix}\end{aligned}$$

  is a group isomorphism.

B. Let G be an abelian group and let

$$H = \left\{a \in G \,|\, a^2 = 1\right\}.$$

be the subset of elements of orders 1 and 2.

- Show that $H \leq G$ is a subgroup of G.
- Give an example of abelian group G, for which the above-defined H is not a cyclic subgroup. *Hint: The first example exists for* $|G| = 4$.