

Math-113, Homework 3, non-textbook problems

A. Consider the set

$$G := \left\{ \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} : a \in \mathbb{Z} \right\}.$$

- Show that it is a group, under matrix multiplication.
- Show that the map

$$\begin{aligned} (\mathbb{Z}, +) &\rightarrow G, \\ a &\mapsto \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \end{aligned}$$

is a group isomorphism.

B. Let G be an abelian group and let

$$H = \{a \in G \mid a^2 = 1\}.$$

be the subset of elements of orders 1 and 2.

- Show that $H \leq G$ is a subgroup of G .
- Give an example of abelian group G , for which the above-defined H is not a cyclic subgroup.