

Math-113, Homework 10, non-textbook problems

A. Let p be a prime and $f(x) \in \mathbb{Z}_p[x]$ be a polynomial in one variable over \mathbb{Z}_p . Prove that for all $t \in \mathbb{Z}_p$, we have

$$(f(t))^p = f(t^p).$$

B. Let p be a prime and suppose that \mathbb{F} is a finite field with p^d elements, where $d \in \mathbb{N}$. Prove that

$$x^{p^d} - x = \prod_{b \in \mathbb{F}} (x - b). \tag{1}$$

where the product is taken over all elements of \mathbb{F} . *Hint: Prove that every $b \in \mathbb{F}$ is a root of the l.h.s. of (1)*