1. Give an example of a power series (centered at \( z_0 = 0 \)) with radius of convergence \( R = 1 \) which converges at \( z = i \) and diverges at \( z = -i \). Justify your answer.

2. Let

\[
f(z) = \sum_{k=0}^{\infty} a_k z^k
\]

be a power series with radius of convergence \( R > 0 \). Suppose that \( f(z) \) satisfies the following differential equation

\[
z^2 f'''(z) + zf'(z) + (z^2 - 1)f(z) = 0 \quad \text{for} \quad |z| < R.
\]

If \( a_1 = \frac{1}{2} \), what should be the value of \( a_5 \)?