SAMPLE MATH 55 MIDTERM 1, SPRING 2014

(1) Mark each of the following questions true (T) or false (F). Provide a sentence or two justifying each answer.

(a) If $x \equiv y \pmod{m}$ then $ax \equiv ay \pmod{m}$.

(b) If $ax \equiv ay \pmod{m}$ then $x \equiv y \pmod{m}$.

(c) The function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = \lfloor \frac{x}{2} \rfloor$ is surjective.

(d) $f(S \cap T) = f(S) \cap f(T)$.

(e) The positive real numbers are countable.

(f) Let $\mathbb{R}$ be the domain, and let $P(x, y)$ be the statement $y^2 = x$. Determine the truth value of the following statement:
$\forall x \exists y\ P(x, y)$.

(2) Prove that if m is a positive integer of the form $4k+3$ for some non-negative integer k, then m is not the sum of the squares of two integers.

(3) Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

(4) Computation.

- Write the number 466 in base 9.

- Does an inverse of 8 $\pmod{75}$ exist? If so, find one.

- Calculate $6^{666} \bmod 23$.

(5) Prove that if p is prime, the only solutions of $x^2 \equiv 1 \pmod{p}$ are integers x such that $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$.

(6) Find all solutions to the system of congruences $x \equiv 2 \pmod 3, x \equiv 1 \pmod 4$, and $x \equiv 3 \pmod 5$.