Math 55 Sample Final Exam

1. (Chapter 1: logic) Inhabitants of an island are three kinds of people: knights who always tell the truth, knaves who always lie, and spies who can either tell the truth or lie. You encounter three people, A, B, and C. You know one of the three people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of the other two is. For the following situation, if possible, determine whether there is a unique solution, list all possible solutions or state that there are no solutions.

   A says "I am not a knight," B says "I am not a spy," and C says "I am not a knave."

2. (Chapter 2) Find the following sums.

   (a)
   $$2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \cdots$$

   (b)
   $$13 + 14 + 15 + \cdots + 132$$

3. (Chapter 4) Prove or disprove the following proposition. Assume that a, b, and m are integers with $m > 1$.

   If $a \equiv b \pmod{m^2}$, then $a \equiv b \pmod{m}$.

4. (Chapter 5) Use the Principle of Mathematical Induction to prove that $3 \mid (n^3 + 3n^2 + 2n)$ for all $n \geq 1$.

5. (Chapter 6) The pigeonhole principle, permutations, and combinations.

   (a) A person giving a party wants to set out 17 assorted cans of soft drinks for his guests. He shops at a store that sells five different types of soft drinks.

      i. How many different selections of cans of 17 soft drinks can he make?

      ii. If root beer is one of the types of soft drink, how many different selections include at least six cans of root beer?

      iii. If the store has only five cans of root beer but at least 17 cans of each other type of soft drink, how many different selections are there?

   (b) Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

6. (Chapter 7) Bayes' Theorem, $E(X)$
   Urn 1 contains 2 blue tokens and 8 red tokens; urn 2 contains 12 blue tokens and 3 red tokens. You pick an urn at random and draw out a token at random from that urn. Given that the token is blue, what is the probability that the token came from urn 1?

7. (Chapter 8) Solve the recurrence relation:
   $$a_n = 5a_{n-1} - 4a_{n-2}, a_0 = 1, a_1 = 0.$$

8. (Chapter 9) equivalence relations
   Suppose that R and S are equivalence relations on a set A. Prove that the relation $R \cap S$ is also an equivalence relation on A.

9. (Chapter 10) isomorphic graphs; Euler path (circuit)

   (a) List all positive integers n such that Q_n has an Euler circuit.

(b) Quiz 8

10. (Chapter 10) graph theory and theorems.

    For the following, either give an example or prove that there are none.

    (a) A simple graph with degrees 1, 2, 2, 3.

    (b) A simple graph with 6 vertices and 16 edges.