Quiz 5, 7/21/16

A space probe near Neptune communicates with Earth using bit strings. Suppose that in its transmissions it sends a 1 one-third of the time and a 0 two-thirds of the time. When a 0 is sent, the probability that it is received correctly is 0.9. When a 1 is sent, the probability that it is received correctly is 0.8.

a. Find the probability that a 0 is received.

b. Use Bayes' theorem to find the probability that a 0 was transmitted, given that a 0 was received.

Solution. Let R_0 be the event that a 0 was received; let S_0 be the event that a 0 was sent; and let S_1 be the event that a 1 was sent. Note that $\bar{S_0} = S_1$. Then we are told that $p(S_1) = \frac{1}{3}, p(S_0) = \frac{2}{3}, p(R_0 \mid S_0) = 0.9$, and $p(R_0 \mid S_1) = 0.2$.

a.

$$\begin{aligned} p(R_0) &= p(R_0 \mid S_0) \cdot p(S_0) + p(R_0 \mid S_1) \cdot p(S_1) \\ &= 0.9 \cdot \frac{2}{3} + 0.2 \cdot \frac{1}{3} \\ &= \frac{1.8 + 0.2}{3} \\ &= \frac{2}{3}. \end{aligned}$$

b. By Bayes' Theorem,

$$\begin{aligned} p(S_0 \mid R_0) &= \frac{p(R_0 \mid S_0) \cdot p(S_0)}{p(R_0 \mid S_0) \cdot p(S_0) + p(R_0 \mid S_1) \cdot p(S_1)} \\ &= \frac{0.9 \cdot \frac{2}{3}}{0.9 \cdot \frac{2}{3} + 0.2 \cdot \frac{1}{3}} \\ &= \frac{0.9 \cdot \frac{2}{3}}{\frac{2}{3}} \\ &= 0.9. \end{aligned}$$

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