Quiz 4, 7/15/16

Suppose a group of 12 people consists of five men and seven women.

    a. How many five-person teams can be chosen that consist of three women and two men?

    b. How many five-person teams contain at least one man?

Solution.   a. By the product rule,

$$
\begin{aligned}
\binom{5}{2} \cdot \binom{7}{3} &= \frac{5!}{2!3!} \cdot \frac{7!}{3!4!} \\
&= \frac{5 \cdot 4 \cdot 3!}{2 \cdot 3!} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 4!} \\
&= 5 \cdot 2 \cdot 7 \cdot 5 \\
&= 350
\end{aligned}
$$

    b. We calculate this number indirectly, buy finding the number of teams that have no man and then subtract from the total.

$$
\begin{aligned}
\binom{12}{5} - \binom{7}{5} &= \frac{12!}{5!(12-5)!} - \frac{7!}{5!(7-5)!} \\
&= \frac{12!}{5!7!} - \frac{7!}{5!2!} \\
&= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 7!} - \frac{7 \cdot 6 \cdot 5!}{5!2} \\
&= 11 \cdot 9 \cdot 8 - 7 \cdot 3 \\
&= 792 - 21 \\
&= 771. \hspace{3cm} \square
\end{aligned}
$$