§Propositional Logic, Predicates, and Equivalency

- A statement or a proposition is a sentence that is true (T) or false (F) but not both.
- The symbol $\neg$ denotes *not*, $\wedge$ denotes *and*, and $\vee$ denotes *or*.
- If p is a proposition, the negation of p, $\neg p$, has opposite truth value from p. If p is true, $\neg p$ is false, if p is false, then $\neg p$ is true.
- If p and q are propositions, the conjunction of p and q, $p \wedge q$, is true when both p and q are true, and is false otherwise.
- If p and q are propositions, the disjunction of p and q, $p \vee q$, is false when both p and q are false, and is true otherwise.
- The symbol $\equiv$ or $\Leftrightarrow$ denotes equivalent truth value.
- The symbol $\rightarrow$ denotes implication. The symbol $\leftrightarrow$ indicates if and only if.
- If propositions p and q are equivalent, they are both true or both false, that is, they both have the same truth value.
- A tautology is a statement that is always true. A contradiction is a statement that is always false.
- *DeMorgan's Laws.*

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

- If p and q are propositions, the conditional "if p then q" (or "p only if q" or "q if p), denoted by $p \rightarrow q$, is false when p is true and q is false; otherwise it is true. p is a sufficient condition for q and q is a necessary condition for p.
- The contrapositive of a conditional statement of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.
- The converse of $p \rightarrow q$ is $q \rightarrow p$.
- The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$.
- If p and q are propositions, the biconditional "p if and only if q," denoted by $p \leftrightarrow q$, is true if both p and q have the same truth values and is false if p and q have opposite truth values. The words *if and only if* are sometimes abbreviated iff.
- Logic formulas

$$p \rightarrow q \equiv \neg p \vee q$$
$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$
$$p \rightarrow q \not\equiv q \rightarrow p$$
$$p \rightarrow q \not\equiv \neg p \rightarrow \neg q$$
$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$
$$\neg(\forall x \text{ in } D, P(x)) \equiv \exists x \text{ in } D \text{ such that } \neg P(x)$$
$$\neg(\exists x \text{ in } D \text{ such that } P(x)) \equiv \forall x \text{ in } D, \neg P(x)$$