3.2 Properties of Determinants

Theorem 3. *Let A be a square matrix.*

1. *If a multiple of one row of A is added to another row to produce a matrix B, then* $\det B = \det A$.
2. *If two rows of A are interchanged to produce B, then* $\det B = -\det A$.
3. *If one row of A is multiplied by k to produce B, then* $\det B = k \cdot \det A$.

Theorem 4. *A square matrix A is invertible if and only if* $\det A \neq 0$.

Theorem 5. *If A is an $n \times n$ matrix, then* $\det A^T = \det A$.

Theorem 6. *If A and B are $n \times n$ matrices, then* $\det AB = (\det A)(\det B)$.

Example 1. *Find the determinant by row reduction to echelon form.* $\begin{vmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{vmatrix}$