14 Partial Derivatives

14.1 Functions of Several Variables

14.1.1 Functions of Two Variables

There are many examples of functions that depend on more than one variable, such as the volume of a cone and the area of a triangle.

Definition 1. *A* function f of two variables *is a rule that assigns to each ordered pair of real numbers* (x, y) *in the domain set, a unique real number* $f(x, y)$.

If we write $z = f(x, y)$, then x and y are independent variables and z is the dependent variable.

Example 1. *A model for the surface area of a human body is given by the function*

$$S = f(w, h) = 0.1091w^{0.425}h^{0.725}$$

where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet.

1. *Find* $f(160, 70)$ *and interpret it.*
2. *What is your own surface area?*

14.1.2 Graphs

Definition 2. *If f is a function of two variables, then the* graph *of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y, z) is in the domain set.*

The graph of a function of one variable is a curve in $\mathbb{R}^2$. The graph of a function of two variables is a surface in $\mathbb{R}^3$.

Example 2. *The function*

$$f(x, y) = ax + by + c$$

is called a linear function. *The graph of f has the equation*

$$z = ax + by + c \Rightarrow ax + by - z + c = 0$$

Thus the graph of a linear function is a plane.

Example 3. *Find the domain and range and sketch the graph of* $h(x, y) = 4x^2 + y^2$.

14.1.3 Level Curves

Definition 3. *The* level curves *(contour curves) of a function of two variables are the curves with equation $f(x, y) = k$, where k is a constant (in the range of f).*

Just as on a contour map, a level curve shows the points that have the same height. Thus the surface is steep where the level curves are close together. The surface is flatter where the level curves are farther apart.

Example 4. *The level curves of the function $f(x, y) = 6 - 3x - 2y$ are straight lines for each value of k in $6 - 3x - 2y = k$. The graph of f is a plane.*

14.1.4 Functions of Three or More Variables

A function $f(x, y, z)$ of three variables is a rule that assigns a unique real number to each ordered triple (x, y, z). It is difficult to visualize a function of three variables by its graph. However, we may examine its *level surfaces*, that is, surfaces with equations $f(x, y, z) = k$, where k is a constant. If the point (x, y, z) moves along a level surface, the value of $f(x, y, z)$ remains fixed.

Example 5. *Describe the level surfaces of the function* $f(x, y, z) = x^2 - y^2 - z^2$.