13.4 Motion in Space: Velocity and Acceleration

When $\vec{r}(t)$ is the position vector of an object at time t, the vector

$$\frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

depicts the average velocity over a time interval of length h. In the limit, we get the (instantaneous) *velocity vector* $\vec{v}(t)$ at time t:

$$\vec{v}(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \vec{r'}(t)$$

Thus the velocity vector is the same as the tangent vector, which points in the direction of the tangent line. The *speed* of the object at time t is the magnitude of the velocity vector. Thus the speed is

$$|\vec{v}(t)| = |\vec{r'}(t)| = \frac{ds}{dt} = \text{rate of change of distance with respect to time}$$

As always, the acceleration is the derivative of the velocity:

$$\vec{a}(t) = \vec{v'}(t) = \vec{r''}(t)$$

Example 1. *Find the velocity, acceleration, and speed of a particle with the position function*

$$\vec{r}(t) = t^2\hat{\imath} + 2t\hat{\jmath} + \ln t\hat{k}$$

Example 2. *Find the velocity and position vectors of a particle that has the acceleration*

$$\vec{a}(t) = \langle \sin(t), 2\cos t, 6t \rangle$$

with initial velocity $\vec{v}(0) = -\hat{k}$ *and initial position* $\vec{r}(0) = \hat{\jmath} - 4\hat{k}$.