6 Applications of Trigonometry

6.1 Transformations of Trigonometric Functions

There are many phenomena that are periodic in nature, such as tides, rider-ship on BART, average temperature at a location, and phases of the moon. Sine and cosine functions are periodic, too. However, the values of sine and cosine varies between 1 and -1. Also, their period is 2π, which are rarely a good fit for the natural phenomena. We need to transform these trigonometric functions to fit the situation. In this section, we revisit transformations from the beginning of the course, this time applied to trigonometric functions.

Amplitude

Example 1. *An example to stretch the cosine or sine function vertically.*

Definition 1. *The* amplitude *of a function is one-half the difference between the maximum and minimum values of the function.*

Remark 1. *Only functions that have maximum and minimum values have amplitude. Thus the tangent function does not have amplitude.*

Remark 2. *A function may have small amplitude and a vertical shift.*

Example 2. *A function that has small amplitude and a vertical shift.*

Period

All trigonometric functions are periodic, meaning that they repeat their behavior at regular intervals:

$$\cos(\theta + 2\pi) = \cos\theta \quad \text{and} \quad \sin(\theta + 2\pi) = \sin\theta$$

Definition 2. *Suppose f is a function and $p > 0$. We say that f has* period p *if p is the smallest positive number such that*

$$f(x + p) = f(x)$$

for every real number x in the domain of f.

A function that has a period is called periodic. Thus the sine and cosine functions have period 2π, whereas the tangent function has a period of π. We may change the period by horizontal stretch.

Example 3. *An example with vertical stretch and horizontal stretch.*

Phase Shift

A *phase shift* is a horizontal shift to the left or to the right.

Example 4. *An example with phase shift.*

Fitting Transformations of Trigonometric Functions to Data

Now we know the tools to fit a regular trigonometric function to model situation.

Example 5. *An example to fit a real world phenomenon with a trigonometric function model.*