

5.6 Addition and Subtraction Formulas

The Cosine of a Sum and Difference

We may obtain the formula for the cosine of a sum of two angles by equating the formula for distance and the formula for the law of cosines:

$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

Example 1. Find an exact expression for cosine of a sum of two famous angles.

If we replace w with the angle $-w$, we obtain the difference formula. Note that cosine is an even and sine is an odd function.

$$\cos(v - w) = \cos v \cos w + \sin v \sin w$$

Example 2. Find an exact expression for cosine of difference between two famous angles.

The Sine of a Sum and Difference

Please consider the following:

$$\begin{aligned}\sin(v + w) &= \cos\left(\frac{\pi}{2} - v - w\right) \\ &= \cos\left(\left(\frac{\pi}{2} - v\right) - w\right) \\ &= \cos\left(\frac{\pi}{2} - v\right) \cos w + \sin\left(\frac{\pi}{2} - v\right) \sin w \\ &= \sin v \cos w + \cos v \sin w.\end{aligned}$$

Thus

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$

Again, if we replace w with the angle $-w$, we obtain the difference formula. Note that cosine is an even and sine is an odd function.

$$\sin(v - w) = \sin v \cos w - \cos v \sin w$$

Example 3. Verify the formula for $\sin\left(\frac{\pi}{2} - \theta\right)$.

The Tangent of a Sum and Difference

We obtain the formula for tangent from the formulas for sine and cosine:

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

Remark 1. You do not need to memorize all six formulas for sum and difference. Just memorize the sum formulas for sine and cosine. The rest will follow.

Example 4. Evaluate $\tan(\pi - \theta)$.

Products of Trigonometric Functions

We may obtain the product formulas from the sum formulas.

Example 5. An example that involves product of trigonometric functions.