4.5 Trigonometry in Right Triangles

Trigonometric Functions via Right Triangles

For any angle on the unit circle, we may drop a line from the endpoint of the corresponding radius to the horizontal axis and form a right triangle. This right triangle has the radius as the hypotenuse and thus the hypotenuse is 1. Any other right triangle with the same acute angle as the reference angle is similar to our right triangle. Thus the ratio of corresponding sides are equal.

Suppose $0 < \theta < \frac{\pi}{2}$. If the second right triangle has legs of sides x and y and hypotenuse r, we have:

$$\cos\theta = \frac{x}{r} \quad \text{and} \quad \sin\theta = \frac{y}{r}.$$

Furthermore,

$$\tan\theta = \frac{y}{x}.$$

We have obtained right-triangle trigonometry:

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$
$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}.$$

These definitions of trigonometric functions in right triangles are very useful in solving problems.

Two Sides of a Right Triangle

Once we know two sides of a right triangle, we can always find the length of the third side by using the Pythagorean Theorem.

Example 1. *Suppose two legs of a right triangle are* 5 *and* 12. *Find the hypotenuse and evaluate* $\sin\theta, \cos\theta,$ *and* $\tan\theta$ *for an acute angle* θ *in this right triangle.*

One Side and One Angle of a Right Triangle

When we know the length of one side and the measure of one acute angle in a right triangle, we can solve the triangle.

Example 2. *Suppose a right triangle has an angle of* 32° *and one leg of* 5. *Solve the triangle.*

We may solve a huge number of real-world problems using trigonometry.

Example 3. *The Earth is approximately a sphere with radius* 3963 *miles. Cleveland has latitude* 41.5° *north. Find the radius of the circle formed by the points with the same latitude as Cleveland.*

Example 4. *Suppose you travel east on the surface of the earth from Cleveland (latitude* 41.5° *north, longitude* 96.8° *west), always staying at the same latitude as Cleveland. You stop when reaching latitude* 41.5° *north, longitude* 75.1° *west (directly north of Philadelphia). How far have you traveled?*

Example 5. *Suppose you need to find the height of a tall building Standing* 20 *meters from the base of the building, you aim a laser pointer at the closest part of the top of the building. You measure that the laser pointer is* 4° *tilted from pointing straight up. The laser pointer is held* 2 *meters above the ground. How tall is the building?*