1.4 Composition of Functions

Definition of Composition

Definition 1. *The composition of functions f and g, denoted by $f \circ g$, is the function*

$$(f \circ g)(x) = f\left(g(x)\right).$$

Note: We first evaluate $g(x)$, then we evaluate $f(g(x))$.

Fact 1. *The domain of $f \circ g$ is the set of numbers x in the domain of g, such that $g(x)$ is in the domain of f.*

Example 1. *Suppose $f(x) = \sqrt{x}, g(x) = \frac{x+1}{x+2}$, and $h(x) = |x-1|$. Evaluate the following.*

a) $(g \circ f)(5)$.

b) $(f \circ h)(-15)$.

c) What is the domain of $g \circ f$?

d) What is the domain of $f \circ h$?

Order Matters in Composition

Example 2. *Suppose that $f(x = x^2 + 1$ and $g(x) = \frac{1}{x}$.*

a) Find $f \circ g$.

b) Find $g \circ f$.

As the above example demonstrates, in general, $f \circ g \neq g \circ f$.

The Identity Function

Consider the following function:

$$I(x) = x.$$

Then

$$I \circ f = f \circ I = f$$

for every function f. We say I is the identity for the operation of composition.

Decomposing Functions

It is usually difficult to start with a function and write it as the composition of two simpler functions.

Example 3. *Suppose*

$$h(x) = \sqrt{\frac{1}{x^2+1} + 2}.$$

a) If $f(x) = \sqrt{x}$, then find a function g such that $h = f \circ g$.

b) If $f(x) = \sqrt{x+2}$, then find a function g such that $h = f \circ g$.