7.3 Area and the Definite Integral

We may approximate the area under a curve with rectangles. If the curve of a function $f(x)$ is above an interval $[a,b]$, we may divide the interval into n subintervals. For simplicity, let each subinterval be the same width:

$$\Delta x = \frac{b-a}{n}$$

The area under the curve is approximately the sum of the areas of rectangles:

$$\text{Area } \approx \sum_i^n f(x_i)\Delta x$$

In the limit, the area becomes *exactly* equal to the sum:

$$\text{Area } = \lim_{n\to\infty} \sum_i^n f(x_i)\Delta x$$

We define the definite integral as this limit:

$$\int_a^b f(x)\ dx = \lim_{n\to\infty} \sum_i^n f(x_i)\Delta x$$

The definite integral gives us the net change.

Example 1. *The total revenue for a product can be calculated as the area under the demand curve. Suppose that he demand curve for a certain wine (in dollars per liter) is*

$$D(q) = \frac{1}{10}q^2 - 10q + 260$$

for $0 \le q \le 50$, where q is the demand in liters. Estimate the total revenue using rectangles of width 10 liters.

Example 2. *Estimate the area under a curve by rectangles.*

Example 3. *In 1987, Canadian Ben Johnson set a world record in the 100-m sprint. The record was later taken away when he was found to have used steroid. His speed at various times was as follows:*

Time (sec)	*Speed (mph)*
0	0
1.84	12.9
7.23	26.3
9.83	25.7

a) Use the information in the table and left endpoints to estimate the distance that Johnson ran in miles. You will first need to calculate Δt for each interval. At the end, you will need to divide by 3600 (the number of seconds in an hour), since the speed is in miles per hour.

b) Repeat part (a), using right endpoints.

c) Wait a second; we know the distance was 100 m. Divide this by 1609, the number of meters in a mile, to find how far Johnson ran in miles. Is your answer from part (a) or (b) closer to the true answer?

Homework

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