

7.2 Substitution

Integration is much more difficult than differentiation. While we may find derivatives rather easily, some integrals are even impossible to find, that is, mathematically proven that they do not exist as a composition of elementary functions!

Example 1. $\int e^{x^2} dx$ does not exist as a composition of elementary functions, so we should not waste our time trying to find such an integral.

Nevertheless, for certain integrals, there exist methods to find their values. The substitution method is one such method. The substitution method uses the idea of differentials.

The differential of a variable is an infinitely small value of that variable. So a differential is a concept rather than a number. However, we may also think of a differential as a small amount of change. The notation for the differential of x is dx . Thus we may have $dx = \Delta x$. If $y = f(x)$, then

$$dy = \frac{dy}{dx} \cdot dx = f'(x)dx$$

The substitution method uses the idea of the chain rule to simplify certain integrals.

$$\int f'(u)u'(x) dx = \int f'(u) du$$

So the idea is that $\int f'(u) du$ is simpler than the original $\int f'(u)u'(x) dx$. Our task is to find $u(x)$ such that $u'(x)$ is present in the integral.

Example 2. The rate of growth of the profit (in millions of dollars) from a new technology is approximated by

$$P'(t) = te^{-t^2},$$

where t represents time measured in years. The total profit in the third year that the new technology is in operation is \$10,000.

- a) Find the total profit function.
- b) What happens to the total amount of profit in the long run?

Example 3. An epidemic is growing in a region according to the rate

$$N'(t) = \frac{100t}{t^2 + 2},$$

where $N(t)$ is the number of people infected after t days.

- a) Find a formula for the number of people infected after t days, given that 37 people were infected at $t = 0$.
- b) Use your answer from part (a) to find the number of people infected after 21 days.

Example 4. A company incurs debt at a rate of

$$D'(t) = 90(t + 6)\sqrt{t^2 + 12t}$$

dollars per year, where t is the amount of time (in years) since the company began. By the fourth year the company had accumulated \$16,260 in debt.

- a) Find the total debt function.
- b) How many years must pass before the total debt exceeds \$40,000?

Homework

§7.2: 41, 45