7 Integration

7.1 Antiderivatives

It is often the case that we may have information about the rate of change of a function, and we want to find out the function itself. For example, we may want to find out the total cost from marginal cost. The reverse operation of find a derivative, is finding the antiderivative.

Definition.

$$F(x) \text{ is an antiderivative of } f(x) \Longleftrightarrow F'(x) = f(x).$$

Is an antiderivative of a function, unique? Let's try an example with simple functions.

Remark. *If $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$, then they differ by a constant:*

$$F(x) - G(x) = C$$

for a constant C.

We use the integral notation to show all antiderivatives of a function:

$$\int f(x)\ dx = F(x) + C, \qquad \text{for arbitrary constant } C$$

We now may have integration rules, similar to differentiation rules we had before. In all such rules, we need an arbitrary constant C for the general antiderivative.

7.1.1 Power Rule in Integration

$$\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$$

Let's try a few power rule examples...

7.1.2 Constant Multiple Rule

$$\int kf(x)\ dx = k\int f(x)\ dx, \quad k = \text{ constant}$$

7.1.3 Sum Rule

For functions $f(x)$ and $g(x)$:

$$\int (f+g)\ dx = \int f\ dx + \int g\ dx$$

Example 1. *What is $\int e^x\ dx$?*

Recall the exception to the power rule, where the power cannot be -1. What if the power *is* -1?

$$\int x^{-1}\ dx = ?$$

Recall that

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

As it turns out, we may have a more general rule in this particular example:

$$\int \frac{1}{x}\ dx = \ln|x| + C$$

Example 2. *Suppose the marginal cost is*

$$C'(x) = 1.2^x(\ln 1.2)$$

and 2 units cost \$9.44. Find the cost function.

Example 3. *Suppose the marginal revenue is*

$$R'(x) = 600 - 5e^{0.0002x}$$

Find the revenue function. Recall that if no items are sold, the revenue is 0.

Example 4. *Suppose* $a(t) = 18t + 8, v(1) = 15, s(1) = 19$. *Find* $s(t)$.

Example 5. *The number of degrees in dentistry (D.D.S or D.M.D) conferred to females in the United States has been increasing steadily in recent decades. Based on data from the National Center for Education Statistics, the rate of change of the number of dentistry degrees can be approximated by*

$$D'(t) = 35.352e^{0.017676t},$$

where t *is the number of years since 1980.*

a) *Find* $D(t)$, *given that 1695 degrees in dentistry were conferred to females in 2001.*

b) *Use the formula from part (a) to project the number of degrees in dentistry that will be conferred to females in 2023.*

Example 6. *The rate of change of the volume* $V(t)$ *of blood in the aorta at time* t *is*

$$V'(t) = -kP(t),$$

where $P(t)$ *is the pressure in the aorta at time* t *and* k *is a constant that depends upon properties of the aorta. The pressure in the aorta is*

$$P(t) = P_0e^{-mt},$$

where P_0 *is the pressure at time* $t = 0$ *and* m *is another constant. Letting* V_0 *be the volume at time* $t = 0$, *find a formula for* $V(t)$.

Homework

§7.1: 47, 55, 61, 65, 71