## 6.4 Implicit Differentiation

So far, every function we have seen have been of the form $y = f(x)$ or equivalent with other letters. This form defines $y$ as a function of $x$, *explicitly*. We may say that $y$ is explicitly defined as a function of $x$. For example, $y = 3x - 2$, or $y = e^{x/2}$.

Sometimes we do not have explicit formula for the function; rather, both $x$ and $y$ are mixed together into an equation that it may be difficult, or even impossible, to solve for $y$. For example, $7x^2 = 5y^2 + 4xy + 1$ or $y \ln x + 2 = x^{3/2}y^{5/2}$. In such situations, we say $y$ is *implicitly* defined as a function of $x$.

Even though $y$ may be implicitly defined as a function of $x$, we may still be able to find $dy/dx$, by differentiating with respect to $x$ on both sides of the equation and applying the chain rule. Then we solve for $dy/dx$.

**Example 1.** *The number of species in a genus, $x$, and the number of genera, $y$, comprising $x$ species, are related by*

$$xy^a = k,$$

*where $a$ and $k$ are constants. Find $dy/dx$.*

**Example 2.** *The demand equation for a certain product is $2p^2 + q^2 = 1600$, where $p$ is the price per unit in dollars and $q$ is the number of units demanded.*

*a) Find and interpret $dq/dp$.*

*b) Find and interpret $dp/dq$.*

**Example 3.** *The equation*

$$f(x)g(N) - m - s(x) = 0$$

*describes the growth rate of phytoplankton at equilibrium, where $x$ is the phytoplankton cell size, $f$ is the maximum growth rate, $N$ is the nutrient concentration, $g$ represents the nutrient limitation experienced by phytoplankton, $m$ is the mortality rate (a constant), and $s$ is the loss due to sinking. Addressing the question of how phytoplankton evolution affects nutrient concentration requires finding the rate of change of $N$ with respect to $x$. Using implicit differentiation, show that*

$$\frac{dN}{dx} = \frac{s'(x) - f'(x)g(N)}{f(x)g'(N)}.$$

**Homework**

§6.4: 43, 47, 53