6.3 Economic Applications

6.3.1 Economic Lot Size

Suppose a company manufactures a constant number of units of a product per year and that the product can be manufactured in several batches of equal size throughout the year.

If the company were to manufacture one large batch every year, it would minimize setup costs but incur high warehouse costs.

If the company were to make many small batches, this would increase setup costs.

Calculus finds the number that should be manufactured in each batch to minimize the total cost. If

$$q = \text{number of units in each batch}$$

and

$$\begin{aligned} k &= \text{cost of storing one unit of the product for one year} \\ f &= \text{fixed setup cost to manufacture the product} \\ g &= \text{cost of manufacturing a single unit of the product} \\ M &= \text{total number of units produced annually} \end{aligned}$$

The the total production cost is:

$$T(q) = \frac{fM}{q} + gM + \frac{kq}{2}$$

which is

$$\left(\text{fixed cost} + \frac{\text{cost}}{\text{unit}} \times \frac{\text{\# units}}{\text{batch}}\right) \frac{\text{\# batches}}{\text{year}} + \text{storage cost} \times \text{\# units in storage}$$

6.3.2 Economic Order Quantity

A company must decide how often to order and how many units to request each time an order is placed. This is called the economic order quantity problem.

If

$$q = \text{number of units to order each time}$$

and

$$\begin{aligned} k &= \text{cost of storing one unit for one year} \\ f &= \text{fixed cost to place an order} \\ M &= \text{total units needed per year} \end{aligned}$$

then

$$\text{Total cost} = \text{Storage cost} + \text{Reorder cost}$$

that is,

$$T = \frac{fM}{q} + \frac{kq}{2}.$$

6.3.3 Elasticity of Demand

The sensitivity of demand to changes in price varies with different items. The demand for essentials, such as milk and fuel, does not change much when the price changes. However, the demand for luxury items, such as jewelry and concert tickets, may change significantly with a small change in price.

One way to measure the sensitivity of demand to changes in price is by the relative change:

$$\frac{\Delta q/q}{\Delta p/p} = \frac{p}{q} \cdot \frac{\Delta q}{\Delta p}$$

We have

$$\lim_{\Delta p \to 0} \frac{\Delta q}{\Delta p} = \frac{dq}{dp}$$

Let $q = f(p)$, where q is demand at a price p. The elasticity of demand is defined as

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

We say:

- the demand is inelastic if $E < 1$
- the demand is elastic if $E > 1$
- the demand has unit elasticity if $E = 1$.

Inelastic means that a small change in price has little effect on demand, while elastic means that a small change in price has more effect on demand.

Total revenue is related to elasticity as follows:

- If demand is inelastic, total revenue increases as price increases.
- If demand is elastic, total revenue decreases as price increases.
- Total revenue is maximized at the price where demand has unit elasticity.

Exercises

§6.3: 9, 13, 27