6.2   Applications of Extrema

To solve maximum and minimum problems, it helps to:

- Read the word problem carefully. You may need to read it a few times until you understand the question.

- If possible, sketch a diagram and label it with known values and unknown variables.

- Write a single-variable function to be maximized or minimized.

- Note the domain of the function.

- Find the critical points in the domain.

- Find maximum and minimum values by comparing the function values at critical points and at end-points, if applicable.

Example 1. *A farmer is constructing a rectangular pen with one additional fence across its width. Find the maximum area that can be enclosed with 2400 m of fencing.*

For most living beings, reproduction is seasonal, that is, it happens only during certain times of the year. For example, whales reproduce every two years during a two-month span. Suppose

$$S = \text{ number of adults present during the reproductive period (spawners}$$

and

$$R = \text{ number of adults that return the next season to reproduce (recruits).}$$

A spawner-recruit or parent-progeny function relates S and R: $R = f(S)$. If $R > S$, then we may harvest

$$H = R - S = f(S) - S$$

individuals. Let S_0 be the number of spawners that will allow the maximum harvest without threatening the population with extinction. Then $H(S_0)$ is called the maximum sustainable harvest.

Example 2. *Find the maximum sustainable harvest if*

$$f(S) = \frac{25S}{S+2}.$$

Homework

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