# 6 Applications of the Derivative

## 6.1 Absolute Extrema

Recall that a local minimum is lower than all the surrounding points. Similarly, a relative maximum is higher than the surrounding points. An absolute maximum is lower than *all* points, not just the surrounding points. For example, Mount Tamalpais is highest point in its vicinity. However, the highest point on Earth is Mount Everest. Similarly, an absolute minimum is the lowest point everywhere.

**Theorem** (Extreme Value Theorem)**.** *A function $f$ that is continuous on a closed interval $[a, b]$ will have both an absolute maximum and an absolute minimum on the interval.*

The Extreme Value Theorem guarantees both absolute maximum and absolute minimum for continuous functions over closed intervals. However, it does not tell us how to find them. To find these absolute extrema, we compare the function value at critical points and at the endpoints. The highest point is the absolute maximum and the lowest point is the absolute minimum.

**Example 1.** *Find relative maxima and minima on a graph. Then find absolute maximum and minimum on that graph and where they occur.*

**Example 2.** *Suppose the total cost of producing $x$ items is*

$$C(x) = 81x^2 + 17x + 324, \quad 1 \leq x \leq 10.$$

*Find the minimum value of the average cost.*

**Example 3.** *For a certain sports utility vehicle,*

$$M(x) = -0.015x^2 + 1.31x - 7.3, \quad 30 \leq x \leq 60,$$

*represents the miles per gallon obtained at a speed of $x$ mph. Find the absolute maximum miles per gallon and the absolute minimum and the speeds at which they occur.*

### Homework

§6.1: 41, 45, 57