

5.4 Curve Sketching

We may apply our knowledge about a function to sketch the graph of that function. This process is called curve sketching. Nowadays we have access to graphing software to graph a function more accurately than by hand. Nevertheless, it is good to know how to graph and how a graph may look like. One reason is that a graphing software has a default viewing window and some important aspects of a graph may be outside of the standard window. Even within the viewing window, a software may miss some properties.

At minimum, we want to look at the derivative to see where the function increases or decreases. To draw better sketch, we can look at concavity (by examining the second derivative), look at intercepts, and investigate potential asymptotes. An asymptote is a straight line that the curve approaches as we move away from the origin.

Example 1. *Graph:*

a) $f(x) = x^4 - 4x^3$

b) $g(x) = 1/(x^2 + 4)$

c) $h(x) = \frac{\ln x^2}{x^2}$

Example 2. *Sketch the graph of a single function that has all of the following properties:*

1. *Continuous for all real numbers*
2. *$f'(x) > 0$ on $(-\infty, -2)$ and $(0, 3)$*
3. *$f'(x) < 0$ on $(-2, 0)$ and $(3, \infty)$*
4. *$f''(x) < 0$ on $(-\infty, 0)$ and $(0, 5)$*
5. *$f''(x) > 0$ on $(5, \infty)$*
6. *$f'(-2) = f'(3) = 0$*
7. *$f'(0)$ does not exist*
8. *Differentiable everywhere except at $x = 0$*
9. *An inflection point at $(5, 1)$*

Homework

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