5 Graphs and Derivatives

5.1 Increasing and Decreasing Functions

In mathematics, the default direction is left-to-right. Moving in this direction, if a graph goes up, we say the function is increasing, and if the graph goes down, we say the function is decreasing. Mathematically, we define increasing and decreasing functions as follows:

$$x_1 < x_2 \iff f(x_1) < f(x_2) \quad \text{means the function is increasing}$$
$$x_1 < x_2 \iff f(x_1) > f(x_2) \quad \text{means the function is decreasing}$$

We may use the derivative to find out whether a function is increasing or decreasing, because derivative is the slope:

$$f'(x) > 0 \iff f \text{ is increasing}$$
$$f'(x) < 0 \iff f \text{ is decreasing}$$
$$f'(x) = 0 \iff f \text{ is constant}$$

To find out the intervals of increasing or decreasing, we locate the places where the derivative is neither positive nor negative, that is, zero or undefined. These locations form the boundaries between increasing and decreasing intervals. We call the points where the derivative is zero or undefined, the critical points.

Thus to find the intervals of increasing or decreasing, we first find the critical points and then check the sign of derivative between critical points.

Example 1. *Suppose the total cost $C(x)$ (in dollars) to manufacture a quantity x of weed killer (in hundreds of liters) is given by*

$$C(x) = x^3 - 2x^2 + 8x + 50.$$

a) Where is $C(x)$ decreasing?

b) Where is $C(x)$ increasing?

Example 2. *Researchers have developed the following function to predict the weight of Holstein cows:*

$$W = 619(1 - 0.905e^{-0.002t})^{1.2386},$$

where W is the weight of the Holstein cow in kilograms that is t days old. Where is this function increasing?

Example 3. *The standard normal probability function is used to describe many different populations. Its graph is called the normal curve. This function is defined as*

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Give the intervals where the function is increasing and decreasing.

Homework

§5.1: 45, 57, 60