

4.5 Derivatives of Logarithmic Functions

If $y = \ln x$, then what is y' ? Consider the following:

$$\begin{aligned}x &= e^y \\ \therefore \frac{dx}{dx} &= \frac{d}{dx}(e^y) \\ \therefore 1 &= e^y \cdot \frac{dy}{dx} \\ \therefore \frac{1}{e^y} &= \frac{dy}{dx} \\ \therefore \frac{1}{x} &= \frac{dy}{dx}.\end{aligned}$$

In the above, we used chain rule to find the derivative. Thus we have the following rule:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0.$$

Note that $\log_b x = \frac{\ln x}{\ln b}$, and hence

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}.$$

Example 1. If the total revenue received from the sale of x items is given by

$$R(x) = 30 \ln(2x + 1),$$

while the total cost to produce x items is $C(x) = x/2$, find the following

- a) The marginal revenue
- b) The profit function $P(x)$
- c) The marginal profit when $x = 60$
- d) Interpret the results of part (c).

Homework

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