

4.2 Product Rule and Quotient Rule

In the previous section we saw the sum rule: $(f + g)' = f' + g'$. Would a similar rule hold for products?

Example 1. Suppose $f(x) = x^2$ and $g(x) = x^3$. Find $f'g'$ and compare to $(fg)'$.

Thus clearly we need a new rule for products.

4.2.1 Product Rule

$$(fg)' = f'g + g'f$$

Example 2. Suppose $f(x) = x^2$ and $g(x) = x^3$. Find $(fg)'$ using the product rule.

Similar to products of functions, the quotient of functions needs its own rule. After all, a quotient is really multiplication by a reciprocal.

4.2.2 Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

4.2.3 Average Cost

If the total cost to manufacture x items is $C(x)$, then the average cost per item is $\bar{C}(x) = C/x$. The marginal average cost is $\bar{C}'(x)$. Similar concept apply for average profit.

Example 3. The total profit (in tens of dollars) from selling x self-help books is

$$P(x) = \frac{5x - 6}{2x + 3}.$$

- a) Find the average profit from selling x books.
- b) Find the marginal average profit function.
- c) Is this a reasonable function for profit? Why or why not?

Homework

§4.2: 39, 43, 53