4 Calculating the Derivative

4.1 Techniques for Finding the Derivative

Using the limit definition to find derivatives is a lengthy process and time consuming, even for simple functions. Starting with today, we will develop formulas to find derivatives and avoid using the definition. We begin with a few notations for the derivative of a function $y = f(x)$:

$$f'(x) = \frac{dy}{dx} = \frac{d}{dx}(f)$$

There are other notations as well that are as widely used as the above notations. The two main notations for the derivative of $y = f(x)$ are $f'(x)$ (read "f prime of x" invented by Newton) and dy/dx (read "dy, dx" invented by Leibniz).

4.1.1 Constant Rule

$$f(x) = k \Longrightarrow f'(x) = 0,$$

where k is a constant. This formula says that the rate of change of a constant is zero, which makes sense, because a constant by definition does not change.

Example 1. *If $f(x) = 7$, then find $f'(x)$.*

4.1.2 Power Rule

$$\frac{d}{dx}(x^k) = kx^{k-1},$$

where k is a constant.

Example 2. *If $g(x) = x^e$, find $g'(x)$.*

4.1.3 Constant Multiple

$$\frac{d}{dx}[kf(x)] = kf'(x),$$

where k is a constant.

Example 3. *If $f(t) = 5t^3$, find $f'(t)$.*

In particular, $\frac{d}{dx}[-f(x)] = -f'(x)$.

4.1.4 Sum Rule

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

Example 4. *If $g(t) = 3t^2 - 8t^5$, find $g'(t)$.*

Example 5. *If a rock is dropped from a 144-ft cliff, is position is given by $s(t) = -16t^2 + 144$ ft, where t is time in seconds since the drop.*

a) What is the velocity 1 second after the drop? 2 seconds after the drop?

b) When will the rock hit the ground?

c) What is the velocity upon impact?

4.1.5 Marginals

In business and economics, they use the word "marginal" to refer to a derivative. Thus marginal cost means derivative of cost and marginal revenue means the derivative of revenue. A marginal is approximately the function value for the next item. We may see this fact by observing that the slope of a tangent line is approximately the change in function for one additional item. For example, $C'(x)/1 \approx C(x+1) - C(x)$.

Example 6. *Assume that the demand is given by* $q = 5000 - 100p$ *and the cost of producing* q *units is* $C(q) = 3000 - 20q + 0.03q^2$. *Find the marginal profit for 1000 units.*

Homework

§4.1: 53, 54, 61, 73, 77