3.2 Continuity

Intuitively, if we can trace a curve without lifting our pencil, we say the curve is continuous. Mathematically, we define a continuous function as follows.

Definition. *We say function $f(x)$ is continuous at $x = c$ if and only if*

$$\lim_{x \to c} f(x) = f(c).$$

Thus for a function to be continuous at $x = c$, we need the limit to exist, the function to exist, and both be equal to each other. If a function is not continuous, we say the function is discontinuous. If a function is continuous at every point of an interval, we say the function is continuous on that interval.

Example 1. *The cost to transport a mobile home depends on the distance, x, in miles that the home is moved. One firm charges as follows:*

Cost per mile	*Distance in miles*
$4.00	$0 < x \leq 150$
$3.00	$150 < x \leq 400$
$2.50	$400 < x$

Find the cost to move a mobile home the following distances

a) 130 miles

b) 150 miles

c) 210 miles

d) Where is the cost function discontinuous?

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3.3 Rates of Change

3.3.1 Average Rate of Change

Example 2. *Suppose a car starts to move at 2 pm and the odometer reads 50 miles. Somewhere during the trip at 5 pm, the odometer reads 170 miles. What was the average speed of the car during this trip?*

Speed is an example of a rate of change, specifically, rate of change of distance with time:

$$speed = \frac{distance}{time}$$

More generally, the average rate of change of a function $f(x)$ over an interval $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a}$$

3.3.2 Instantaneous Rate of Change

The car in the previous example may not have traveled at constant speed, rather, sometimes stuck in traffic or behind stop light, and sometimes going faster on the freeway. To obtain instantaneous rate of change at some specific point, we may calculate the average speed close to our point. This process would remove all the extra delays or speed-ups that occur elsewhere.

Definition. *The instantaneous rate of change of a function $f(x)$ at $x = a$ is*

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}.$$

From now on, if we just say "rate of change," we mean instantaneous rate of change, because we will work with instantaneous rates of change for the most part.

Example 3. *The revenue, in thousands of dollars, from selling x items is*

$$R = 10x - 0.002x^2.$$

a) *Find the average rate of change of revenue when the sale is increased from 1000 to 1001 items.*

b) *Find and interpret the (instantaneous) rate of change of revenue when 1000 items are sold. This number is called the marginal revenue at $x = 1000$.*

c) *Find the additional revenue if the sale is increased from 1000 to 1001 items.*

d) *Compare the answers from parts (a) and (c). What do you notice? How do these answers compare with your answer to part (b)?*

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