13.2 Derivatives of Trigonometric Functions

Recall the basic trigonometric identities. For every angle t:

$$\sin^2 t + \cos^2 t = 1$$
$$\tan t = \frac{\sin t}{\cos t}.$$

There are many more trigonometric identities that we do not study in this course. We may prove the formulas for derivatives of trigonometric functions using trigonometric identities and the limit definition of derivative, as well as the following important limit:

$$\lim_{t\to 0} \frac{\sin t}{t} = 1.$$

Following are derivative rules for trigonometric functions:

$$\frac{d}{dt}(\sin t) = \cos t$$
$$\frac{d}{dt}(\cos t) = -\sin t$$
$$\frac{d}{dt}(\tan t) = \sec^2 t.$$

Derivatives of the other 3 functions are derived similarly:

$$\frac{d}{dt}(\sec t) = \sec t \tan t$$
$$\frac{d}{dt}(\csc t) = -\csc t \cot t$$
$$\frac{d}{dt}(\cot t) = -\csc^2 t.$$

Example 1. *The amount of electricity consumption (in trillion BTUs) in U.S. in 2013 is modeled by*

$$C(t) = 70.25 \sin(1.012t + 0.4761) + 397.3,$$

where $t = 1$ corresponds to January.

a) Find $C'(t)$.

b) Find and interpret $C'(3)$.

Example 2. *A thief tries to enter a building by placing a ladder over a 9-ft-high fence so it rests against the building, which is 2 ft back from the fence. What is the length of the shortest ladder that can be used? [Hint: Let θ be the angle between the ladder and the ground. Express the length of the ladder in terms of θ, and then find the value of θ that minimizes the length of the ladder.]*

Homework

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