12.7 L'Hôpital's Rule

When we try to evaluate limits, we frequently encounter limits of the form $0/0, \infty/\infty, 1^\infty, 0 \times \infty, \infty - \infty$, and ∞^0. These are called indeterminate forms, because they can be any number. For example, $0/0 = x$ is a true statement for every number x, because the multiplication equivalent equation is $0 = 0 \cdot x$, which is true for every number x. For instance, $0/0 = 3$ is true, as well as $0/0 = -\pi$.

When we have indeterminate forms, we have to find some way to determine the limit, that is, find that particular number.

L'Hôpital's rule is useful for indeterminate forms $0/0$ and ∞/∞.

Theorem (L'Hôpital's Rule)**.** *If* $\lim_{x\to a} \frac{f(x)}{g(x)}$ *is of indeterminate form* $0/0$ *or* ∞/∞*, then*

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limits exist.

Note that L'Hôpital's rule also works for limits at infinity.

Example 1. *Find the limits.*

a) $\lim_{x\to 3} \frac{x^3+x^2-11x-3}{x^2-3x}$

b) $\lim_{x\to 0} \frac{\sqrt{3-x}-\sqrt{3+x}}{x}$

c) $\lim_{x\to 0} \frac{e^x}{8x^5-3x^4}$

d) $\lim_{x\to 0} \frac{xe^{-x}}{2e^{2x}-2}$

e) $\lim_{x\to 0} \left(\frac{12e^x}{x^3} - \frac{12}{x^3} - \frac{12}{x^2} - \frac{6}{x}\right)$

Homework

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