1 Linear Functions

1.1 Slopes and Equations of Lines

Suppose we use the $x - y$ plane with x as the horizontal axis and y as the vertical axis, to draw graphs.

We indicate how much a straight line is slanted by a number called slope. The slope of a line is a ratio:

$$m = \frac{\Delta y}{\Delta x}$$

where m is the slope, y is the change in y, and Δx is the change in x.

To write the equation of a line, the slope is the most important number. We also need another information to write the equation of a line completely. Usually, we use the y-intercept, which is where the line intersects the vertical axis. Thus we may write the equation of a line with slope m and y-intercept b as

$$y = mx + b$$

For a horizontal line, $\Delta y = 0$, so the slope is zero. Thus the equation of a horizontal line is of the form $y = k$, where k is a constant. For a vertical line, $\Delta x = 0$, so the slope is undefined, because division by zero is undefined. The equation of a vertical line is $x = k$, where k is a constant (x is fixed).

Example 1. *(1.1.68) A study found that the peak vertical force on a trotting Shetland pony increased linearly with the pony's speed, and that when the force reached a critical level, the pony switched from a trot to a gallop. For one pony, the critical force was 1.16 times its body weight. It experienced a force of 0.75 times its body weight at a speed of 2 meters per second and a force of 0.93 times its body weight at 3 meters per second. At what speed did the pony switch from a trot to a gallop?*

1.2 Linear Functions and Applications

We may write the relationship between two variables such that one variable is a function of the other variable. Usually, we have y as a function of x. The notation is $y = f(x)$. Depending on what x is, y changes accordingly. We say y depends on x, so x is the independent variable and y is the dependent variable. If x and y are related linearly, we may write

$$y = f(x) = mx + b$$

which is the same equation of a straight line we saw before.

1.2.1 Supply and Demand

The supply is the quantity of a certain good that producers are willing to provide. The demand is the quantity of the good that consumers are willing to buy. Usually, as the price of an item (p) increases, the demand (the quantity q) decreases, because fewer people buy more expensive items. And as the price p decreases, q increases. If the relationship between p and q is linear, the slope must be negative for a demand function.

If we view from the perspective of the seller, they want to maximize their revenue. So the seller wants to sell more items at higher prices. As the supply (the quantity q) increases, the price of each item p also increases. For a linear relationship, the slope between p and q is positive for a supply function.

The point where the supply and demand are equal is called the equilibrium. If the price is higher than the equilibrium the supplier would have a surplus of items. If the price is lower than the equilibrium, there would be a shortage of the product. Graphically, the equilibrium is the point where the supply function intersects the demand function.

Example 2. *(1.2.30) Suppose the supply and demand functions are as follows:*

$$p = 1.4q - 0.6$$
$$p = -2q + 3.2$$

where p is the price per pound and q is the quantity in thousands of pounds.

a) Which function is demand and which one is the supply function?

b) Find the equilibrium point.

1.2.2 Cost

We may consider the cost of producing a product as a fixed cost plus a variable cost.

$$C(x) = C_k + C_v(x)$$

The fixed cost, as the name implies, is fixed and does not change if the number of items x changes. The variable cost C_v varies as x varies, so the variable cost depends on how many items x we produce.

Business and economics have their own lingo. For instance, the rate of change of cost C with the quantity x is called the marginal cost. The rate of change of cost is the slope of the cost function.

1.2.3 Revenue

Revenue is the money a company receives from selling their product. How do we calculate a revenue? Consider the following example.

Example 3. *If the price of each T-shirt is \$5 and we sell 8 T-shirts, what is our revenue?*

1.2.4 Profit

Profit is the money a company makes after paying their costs. How do we calculate profit from revenue and cost? Consider the following example.

Example 4. *Suppose the revenue from selling T-shirts is \$40 and the cost of producing those T-shirts is \$35. What is the profit?*

1.2.5 Break-Even

Break-even happens when a company does not make a profit and does not lose money either. This means the profit is zero.

Example 5. *(1.2.34) To produce x units of a medal costs $C(x) = 12x + 39$ dollars. We sell each medal at \$25.*

a) Find the break-even quantity.

b) Find the profit from selling 250 medals.

Homework

§1.1: 62
§1.2: 29, 33