

## Solutions Worksheet 3

1) See book for explanation

2) a) AB has slope  $\frac{3-7}{2-1} = -4 \Rightarrow H_c$  has slope  $\frac{1}{4}$

BC has slope  $\frac{4-3}{5-2} = \frac{1}{3} \Rightarrow H_A$  has slope  $-3$

CA has slope  $\frac{4-7}{5-1} = -\frac{3}{4} \Rightarrow H_B$  has slope  $\frac{4}{3}$

$$\Rightarrow H_A: y-7 = -3(x-1) \Rightarrow y = -3x + 10$$

$$H_B: y-3 = \frac{4}{3}(x-2) \Rightarrow y = \frac{4}{3}x + \frac{1}{3}$$

$$H_C: y-4 = \frac{1}{4}(x-5) \Rightarrow y = \frac{1}{4}x + \frac{11}{4}$$

b) intersection of  $H_A$  and  $H_B$ :

$$-3x + 10 = \frac{4}{3}x + \frac{1}{3}$$

$$\Rightarrow \frac{29}{3} = \frac{13}{3}x$$

$$\Rightarrow x = \frac{29}{13}$$

$$\Rightarrow y = -3\left(\frac{29}{13}\right) + 10 = \frac{43}{13}$$

intersection of  $H_B$  and  $H_C$ :

$$\frac{4}{3}x + \frac{1}{3} = \frac{1}{4}x + \frac{11}{4}$$

$$\left(\frac{4}{3} - \frac{1}{4}\right)x = \left(\frac{11}{4} - \frac{1}{3}\right)$$

$$\frac{13}{12}x = \frac{25}{12}$$

$$\Rightarrow x = \frac{25}{13}$$

$$\Rightarrow y = \frac{1}{4}\left(\frac{25}{13}\right) + \frac{11}{4} = \frac{43}{13}$$

intersection of  $H_A$  and  $H_C$

$$-3x + 10 = \frac{1}{4}x + \frac{11}{4}$$

$$\Rightarrow \frac{13}{4}x = \frac{25}{4}$$

$$\Rightarrow x = \frac{25}{13}$$

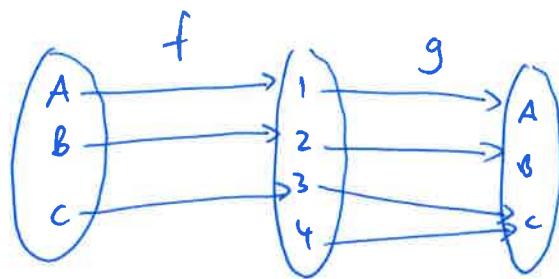
$$\Rightarrow y = -3\left(\frac{25}{13}\right) + 10 = \frac{43}{13}$$

Hence  $H_A$ ,  $H_B$  and  $H_C$  intersect in one point  $(\frac{25}{13}, \frac{43}{13})$

c) No, it is not a coincidence. The altitudes (in our case  $H_A$ ,  $H_B$  and  $H_C$ ) of a triangle always intersect in one point, namely the orthocenter of the triangle.

3)

1) No!

2) Yes! Assume  $x_1$  and  $x_2$  in  $X$  such that

$$f(x_1) = f(x_2).$$

$$\text{Then } g(f(x_1)) = g(f(x_2))$$

$$\Rightarrow (g \circ f)(x_1) = (g \circ f)(x_2)$$

$$\Rightarrow I(x_1) = I(x_2)$$

$$\Rightarrow x_1 = x_2$$

$\Rightarrow f$  is injective.

3) Yes. Let  $x \in X$ . Then  $g(f(x)) = x$ 

Hence  $g$  is surjective.

4) No! See above example from 1).